**The Open University of Sri Lanka**

**B.Sc/B.Ed. Degree Programme – Level 04**

**Open Book Test (OBT) - 2015/2016**

**Pure Mathematics**

**PUU2144/PUE4144 – Group Theory I**

**Duration: - One Hour.**

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**Date: -08.10.2016** **Time: - 2.30 p.m. – 3.30 p.m.**

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**Answer All Questions.**

1. (a) Let + and $\cdot$ be the usual binary operations of addition and multiplication on the set $\mathbb{Z}$ and let $H = \{n^2 \mid n \in \mathbb{Z}^+\}$. Determine whether $H$ is closed under *addition* and *multiplication.*

   (b) Examine the *Commutativity* and *Associativity* for the following binary operations.

   (i) On $\mathbb{Q}$, define binary operation $*$ such that $a, b \in \mathbb{Q}$, $a * b = \frac{ab}{2}$.

   (ii) On $\mathbb{Z}^+$, define binary operation $*$ such that $a, b \in \mathbb{Z}^+$, $a * b = a^b$.

   (c) Which of the following sets have identity and inverse elements? Justify your answer.

   (i) The set $(\mathbb{R}, \times)$ under the usual operation of multiplication.

   (ii) The set $(\mathbb{Z} \setminus \{-1\}, *)$(All integers except -1) under the binary operation defined by $a * b = 1 + a + b + ab$.

2. (a) Find all subgroups of $\mathbb{Z}_{10}$ and draw the corresponding lattice diagram.

   (b) Let $(G, \times)$ be a group (The opreation $\times$ is a usual multiplication). If $H < G$, prove that $gH = H$ if and only if $g \in H$

   (c) The permutation $\sigma$ is given by $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 5 & 4 & 6 & 2 & 1 & 7 & 3 \end{pmatrix}$, write down $\sigma$ in the (disjoint) cyclic notation.
   State whether that $\sigma$ is even or odd permutation.

   What is the Order of $\sigma$ ?