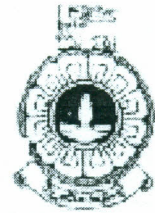


The Open University of Sri Lanka
B.Sc/B.Ed. Degree Programme – Level 04
Open Book Test (OBT) - 2015/2016
Pure Mathematics
PUU2144/PUE4144 – Group Theory I
Duration: - One Hour.



Date: -08.10.2016

Time: - 2.30 p.m. – 3.30 p.m.

Answer All Questions.

1. (a) Let $+$ and \cdot be the usual binary operations of addition and multiplication on the set \mathbb{Z} and let $H = \{n^2 \mid n \in \mathbb{Z}^+\}$. Determine whether H is closed under *addition* and *multiplication*.
- (b) Examine the *Commutativity* and *Associativity* for the following binary operations.
- (i) On \mathbb{Q} , define binary operation $*$ such that $a, b \in \mathbb{Q}$, $a * b = \frac{ab}{2}$.
- (ii) On \mathbb{Z}^+ , define binary operation $*$ such that $a, b \in \mathbb{Z}^+$, $a * b = a^b$.
- (c) Which of the following sets have identity and inverse elements? Justify your answer.
- (i) The set (\mathbb{R}, \times) under the usual operation of multiplication.
- (ii) The set $(\mathbb{Z} \setminus \{-1\}, *)$ (All integers except -1) under the binary operation defined by $a * b = 1 + a + b + ab$.
2. (a) Find all subgroups of \mathbb{Z}_{10} and draw the corresponding lattice diagram.
- (b) Let (G, \times) be a group (The operation \times is a usual multiplication). If $H < G$, prove that $gH = H$ if and only if $g \in H$
- (c) The permutation σ is given by $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 5 & 4 & 6 & 2 & 1 & 7 & 3 \end{pmatrix}$, write down σ in the (disjoint) cyclic notation.
State whether that σ is even or odd permutation.
What is the Order of σ ?