

The Open University of Sri Lanka
B.Sc/B.Ed. Degree Programme
Open Book Test (OBT) - 2015/2016
Pure Mathematics - Level 05
PUU3143/PUE5143 Riemann Integration



Duration: One Hour

Date: 01.10.2016

Time: 04.00pm - 05.00pm

Answer ALL questions.

1. Let

$$f(x) = \begin{cases} 1, & 0 \leq x < \frac{1}{2}; \\ 0, & \frac{1}{2} \leq x \leq 1, \end{cases} \quad g(x) = \begin{cases} 0, & 0 \leq x \leq \frac{1}{2}; \\ 1, & \frac{1}{2} < x \leq 1, \end{cases}$$

be two functions defined on a closed interval $[0, 1]$. Let $P_1 = \left\{0, \frac{1}{4}, \frac{3}{4}, 1\right\}$ and $P_2 = \left\{0, \frac{1}{4}, \frac{1}{2}, 1\right\}$ are the partitions of $[0, 1]$. Evaluate each of the following:

(a) $U(P_1 * P_2, f)$;

(b) $U(P_1 * P_2, g)$;

(c) $\int_0^1 f(x) dx$;

(d) $\int_0^1 g(x) dx$.

2. Let $h(x)$ be a function on a closed interval $[0, 1]$ and is defined as follows

$$h(x) = \begin{cases} 3, & 0 \leq x < \frac{1}{3}; \\ 2, & x = \frac{1}{3}; \\ 1, & \frac{1}{3} < x \leq \frac{2}{3}; \\ 4, & \frac{2}{3} < x \leq 1. \end{cases}$$

Show that h is Riemann integrable on $[0, 1]$ and $\int_0^1 h(x) dx = \frac{8}{3}$.