The Open University of Sri Lanka
B.Sc/B.Ed. Degree Programme
No Book Test (NBT) - 2015/2016
Pure Mathematics - Level 04
PUU2140/PUE4140– Sequences and Series

Duration: - One hour

Date: - 14-05-2016 **Time: - 2:30pm – 3:30pm**

Answer all questions

(01)

(i) Find a sequence $< a_n >$ of real numbers such that for each $k \in \mathbb{N}$, $\{n \in \mathbb{N} : a_n = k\}$ is infinite.

(ii) Let $b_n = \sin\left(\frac{n\pi}{2}\right)$ for each $n \in \mathbb{N}$. Find $L(b_n)$ where

$$L(b_n) = \left\{l : \text{there exists a subsequence} < b_{n_k} > \text{of} < b_n > \text{such that} \lim_k b_{n_k} = l\right\}.$$

(iii) Let $< c_n >$ be a bounded sequence of real numbers. Prove that $\liminf c_n \le \limsup c_n$.

(iv) Prove that there exists a sequence $< x_n >$ of real numbers such that $\liminf x_n = 2016$ and $\limsup x_n = 6102$.

(02)

(i) Find a series $\sum_{n=1}^{\infty} x_n$ such that $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + \cdots + x_n + \cdots$ diverges,

$(x_1 + x_2) + (x_3 + x_4) + (x_5 + x_6) + (x_7 + x_8) + \cdots + (x_{2n-1} + x_{2n}) + \cdots = 24$ and

$x_1 + (x_2 + x_3) + (x_4 + x_5) + (x_6 + x_7) + \cdots + (x_{2n} + x_{2n+1}) + \cdots = 36$.

(ii) Discuss the convergence of each of the following series:

(a) $\sum_{n=1}^{\infty} \frac{n+2}{n^3+2}$ (b) $\sum_{n=1}^{\infty} \frac{3n+2}{4n+5}$ (c) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$

(d) $\sum_{n=1}^{\infty} \frac{n^n}{n!}$ (e) $\sum_{n=1}^{\infty} \left(1 - \frac{1}{n}\right)^{n^2}$