

The Open University of Sri Lanka
B.Sc/B.Ed. Degree Programme
Continuous Assessment Test (OBT) - 2015/2016
Pure Mathematics - Level 04
PUU2141/PUE4141- Continuous Functions



Duration: - One hour

Date: - 10-04-2016

Time: - 2:30pm – 3.30pm.

Answer all questions

(01)

- (i) Prove that the set of limit points of $(0,1) \cap \mathbb{Q}$ is equal to $[0,1]$.
- (ii) Construct a bounded set of real numbers with exactly 3 limit points.
- (iii) Let $f(x) = 7x + 2, x \in \mathbb{R}$. Prove that $\lim_{x \rightarrow -1} f(x) = -5$.
- (iv) Let $f(x) = x^2 + 1, x \in \mathbb{R}$. Prove that $\lim_{x \rightarrow 1} f(x) \neq 3$.

(02)

- (i) Find two subsets I_1 and I_2 of \mathbb{R} such that $I_1 \cap I_2 = \Phi$ but 0 is a limit point of both I_1 and I_2 .
- (ii) Define two functions f and g on \mathbb{R} into \mathbb{R} such that $\lim_{x \rightarrow 0} f(x)$, $\lim_{x \rightarrow 0} g(x)$ does not exist, but $\lim_{x \rightarrow 0} f(x) + g(x)$ exist.
- (iii) Let $f(x) = \begin{cases} 1 & ; x \in \mathbb{Q} \\ -1 & ; x \in \mathbb{R} \setminus \mathbb{Q} \end{cases}$. Prove that $\lim_{x \rightarrow \infty} f(x)$ does not exist.
- (iv) Define two functions $f, g: (0,1) \rightarrow \mathbb{R}$ such that $\lim_{x \rightarrow 0^+} f(x), \lim_{x \rightarrow 0^+} g(x)$ does not exist but $\lim_{x \rightarrow 0^+} f(x)g(x) = 0$.