

The Open University of Sri Lanka
B.Sc/B.Ed. Degree Programme
No Book Test (NBT) - 2015/2016
Pure Mathematics - Level 03
PUU1140/PUE3140- Logic & Mathematical Proofs



Duration: - One hour

Date: - 07-05-2016

Time: - 9.00am – 10.00am

01. Prove each of the following statements:

(i) For each $x, y \in \mathbb{N}$, $\frac{x+3y}{x+y} < \sqrt{3} < \frac{x}{y}$ or $\frac{x+3y}{x+y} > \sqrt{3} > \frac{x}{y}$.

(ii) $27^4 + 1$ is not a prime and $27^4 + 4$ is not a prime.

(iii) For each $x \in \mathbb{R}$, if $x = \sqrt{x+6}$ then $x = 3$ or $x = -2$.

(iv) For each $x, y \in \mathbb{N}$, $x + y$ is odd, implies x is odd or y is odd.

(v) For each $n \in \mathbb{N}$, $\sum_{k=1}^n k = \frac{n}{3}$ if and only if $\sum_{k=1}^n k^2 = \frac{n^3}{3}$.

(\mathbb{R} is the set of all real numbers and \mathbb{N} is the set of all positive integers.)

02. Prove each of the following statements:

(i) There exist $x, y \in \mathbb{R}$ such that $x - y = 10^{-133}$ and $x^2 - y^2 = 10^{133}$.

(ii) For each $x > 0$, $x^3 + 91 \geq 36x$.

(iii) For each $n \in \mathbb{N}$, $(3 + \sqrt{5})^n + (3 - \sqrt{5})^n \in \mathbb{N}$.

(Hint: Use induction)