THE OPEN UNIVERSITY OF SRI LANKA
B.Sc. /B.Ed. Degree Programme
APPLIED MATHEMATICS-LEVEL 05
APU3146/APE5146 - Operations Research
NO BOOK TEST 2016/2017
Duration: One Hour

Date: 05.11.2017	**Time: 10.30 a.m- 11.30 a.m**

Answer all questions

(1) A supermarket has two girls serving at the counters. The customers arrive in a Poisson distribution at the rate of 12 per hour. The service time for each customer is exponential with mean 6 minutes.

(i) Find the probability that an arriving customer has to wait for service.
(ii) Find the average number of customers in the system.
(iii) What is the average waiting time of an arriving customer?

(2) A car servicing station has two bags where service can be offered simultaneously. Due to space limitation, only four cars are accepted for servicing. The arrival pattern is Poisson with a mean of one car every minute during the peak hours. The service time is exponential with mean 6 minutes. Find

(a) The average number of cars in the service station,
(b) The average number of cars in the system during the peak hours.

Formulas (in the usual notation)

(M/M/C):(∞/FIFO) Queuing System

$$P_n = \begin{cases} \frac{1}{n!}\rho^n P_0 & ;1 \le n \le C \\ \frac{1}{C^{n-C}C!}\rho^n P_0 & ;n > C \end{cases}$$

$$E(m) = \frac{\lambda\mu\left(\frac{\lambda}{\mu}\right)^C P_o}{(C-1)!(C\mu-\lambda)^2} \qquad E(n) = E(m) + \frac{\lambda}{\mu}$$

$$P_0 = \left[\sum_{n=0}^{C-1}\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{C!}\left(\frac{\lambda}{\mu}\right)^C \frac{C\mu}{C\mu-\lambda}\right]^{-1}$$

$$E(w) = \frac{1}{\lambda}E(m) \qquad E(v) = E(w) + \frac{1}{\mu}$$

(M/M/C): (N/FIFO) Model

$$P_n = \begin{cases} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n P_0 & ;0 \le n \le C \\ \frac{1}{C^{n-1}C!}\left(\frac{\lambda}{\mu}\right)^n P_0 & ;C < n \le N \end{cases}$$

$$P_0 = \begin{cases} \left[\sum_{n=0}^{C-1}\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{C!}\left(\frac{\lambda}{\mu}\right)^C \left\{1-\left(\frac{\lambda}{C\mu}\right)^{N-C+1}\right\}\frac{C\mu}{C\mu-1}\right]^{-1} & ;\frac{\lambda}{C\mu} \ne 1 \\ \left[\sum_{n=0}^{C-1}\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{C!}\left(\frac{\lambda}{\mu}\right)^C (N-C+1)\right]^{-1} & ;\frac{\lambda}{C\mu} = 1 \end{cases}$$

$$E(m) = \frac{P_o (C\rho)^C \rho}{C!(1-\rho)^2}\left[1-\rho^{N-C+1} - (1-\rho)(N-C+1)\rho^{N-C}\right] \qquad E(w) = E(v) - \frac{1}{\mu}$$

$$E(n) = E(m) + C - P_0\sum_{n=0}^{C-1}\frac{(C-n)(\rho C)^n}{n!} \qquad E(v) = {[E(n)]}\big/{\lambda'}, \text{ where } \lambda' = \lambda(1-P_N)$$

(M/M/R):(K/GD) Model

$$P_n = \begin{cases} \binom{K}{n}\left(\frac{\lambda}{\mu}\right)^n P_0 & ;\ 0 \le n < R \\ \binom{K}{n}\frac{n!}{R^{n-R}R!}\left(\frac{\lambda}{\mu}\right)^n P_0 & ;\ R \le n \le K \end{cases}$$

$$P_0 = \left[\sum_{n=0}^{R-1}\binom{K}{n}\left(\frac{\lambda}{\mu}\right)^n + \sum_{n=R}^{K}\binom{K}{n}\frac{n!}{R^{n-R}R!}\left(\frac{\lambda}{\mu}\right)^n\right]^{-1}$$

$$E(n) = P_0\left[\sum_{n=0}^{R-1} n\binom{K}{n}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{R!}\sum_{n=R}^{K} n.\binom{K}{n}\frac{n!}{R^{n-R}}\left(\frac{\lambda}{\mu}\right)^n\right] \qquad E(v) = \frac{E(n)}{\lambda[K-E(n)]}$$

$$E(m) = \sum_{n=R}^{K}(n-R)P_n \qquad E(w) = \frac{E(m)}{\lambda[K-E(n)]}$$

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