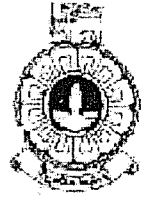


THE OPEN UNIVERSITY OF SRI LANKA
 B.Sc. /B.Ed. Degree Programme
 APPLIED MATHEMATICS-LEVEL 05
 APU3146/APE5146 - Operations Research
 NO BOOK TEST 2016/2017
Duration: One Hour



Date: 05.11.2017

Time: 10.30 a.m- 11.30 a.m

Answer all questions

- (1) A supermarket has two girls serving at the counters. The customers arrive in a Poisson distribution at the rate of 12 per hour. The service time for each customer is exponential with mean 6 minutes.
- Find the probability that an arriving customer has to wait for service.
 - Find the average number of customers in the system.
 - What is the average waiting time of an arriving customer?
- (2) A car servicing station has two bays where service can be offered simultaneously. Due to space limitation, only four cars are accepted for servicing. The arrival pattern is Poisson with a mean of one car every minute during the peak hours. The service time is exponential with mean 6 minutes. Find
- The average number of cars in the service station,
 - The average number of cars in the system during the peak hours.

Formulas (in the usual notation)

(M/M/C):(∞/FIFO) Queuing System

$$P_n = \begin{cases} \frac{1}{n!} \rho^n P_0 & ; 1 \leq n \leq C \\ \frac{1}{C^{n-C} C!} \rho^n P_0 & ; n > C \end{cases}$$

$$E(m) = \frac{\lambda \mu \left(\frac{\lambda}{\mu} \right)^C P_0}{(C-1)!(C\mu - \lambda)^2} \quad E(n) = E(m) + \frac{\lambda}{\mu}$$

$$P_0 = \left[\sum_{n=0}^{C-1} \frac{1}{n!} \left(\frac{\lambda}{\mu} \right)^n + \frac{1}{C!} \left(\frac{\lambda}{\mu} \right)^C \frac{C\mu}{C\mu - \lambda} \right]^{-1}$$

$$E(w) = \frac{1}{\lambda} E(m) \quad E(v) = E(w) + \frac{1}{\mu}$$

(M/M/C): (N/FIFO) Model

$$P_n = \begin{cases} \frac{1}{n!} \left(\frac{\lambda}{\mu}\right)^n P_0 & ; 0 \leq n \leq C \\ \frac{1}{C^{n-C} C!} \left(\frac{\lambda}{\mu}\right)^n P_0 & ; C < n \leq N \end{cases}$$

$$P_0 = \begin{cases} \left[\sum_{n=0}^{C-1} \frac{1}{n!} \left(\frac{\lambda}{\mu}\right)^n + \frac{1}{C!} \left(\frac{\lambda}{\mu}\right)^C \left\{ 1 - \left(\frac{\lambda}{C\mu}\right)^{N-C+1} \right\} \frac{C\mu}{C\mu-1} \right]^{-1} & ; \frac{\lambda}{C\mu} \neq 1 \\ \left[\sum_{n=0}^{C-1} \frac{1}{n!} \left(\frac{\lambda}{\mu}\right)^n + \frac{1}{C!} \left(\frac{\lambda}{\mu}\right)^C (N-C+1) \right]^{-1} & ; \frac{\lambda}{C\mu} = 1 \end{cases}$$

$$E(m) = \frac{P_0 (C\rho)^C \rho}{C!(1-\rho)^2} [1 - \rho^{N-C+1} - (1-\rho)(N-C+1)\rho^{N-C}] \quad E(w) = E(v) - \frac{1}{\mu}$$

$$E(n) = E(m) + C - P_0 \sum_{n=0}^{C-1} \frac{(C-n)(\rho C)^n}{n!} \quad E(v) = \frac{[E(n)]}{\lambda'}, \text{ where } \lambda' = \lambda(1-P_N)$$

(M/M/R): (K/GD) Model

$$P_n = \begin{cases} \binom{K}{n} \left(\frac{\lambda}{\mu}\right)^n P_0 & ; 0 \leq n < R \\ \binom{K}{n} \frac{n!}{R^{n-R} R!} \left(\frac{\lambda}{\mu}\right)^n P_0 & ; R \leq n \leq K \end{cases}$$

$$P_0 = \left[\sum_{n=0}^{R-1} \binom{K}{n} \left(\frac{\lambda}{\mu}\right)^n + \sum_{n=R}^K \binom{K}{n} \frac{n!}{R^{n-R} R!} \left(\frac{\lambda}{\mu}\right)^n \right]^{-1}$$

$$E(n) = P_0 \left[\sum_{n=0}^{R-1} n \binom{K}{n} \left(\frac{\lambda}{\mu}\right)^n + \frac{1}{R!} \sum_{n=R}^K n \binom{K}{n} \frac{n!}{R^{n-R}} \left(\frac{\lambda}{\mu}\right)^n \right] \quad E(v) = \frac{E(n)}{\lambda[K - E(n)]}$$

$$E(m) = \sum_{n=R}^K (n-R) P_n \quad E(w) = \frac{E(m)}{\lambda[K - E(n)]}$$
