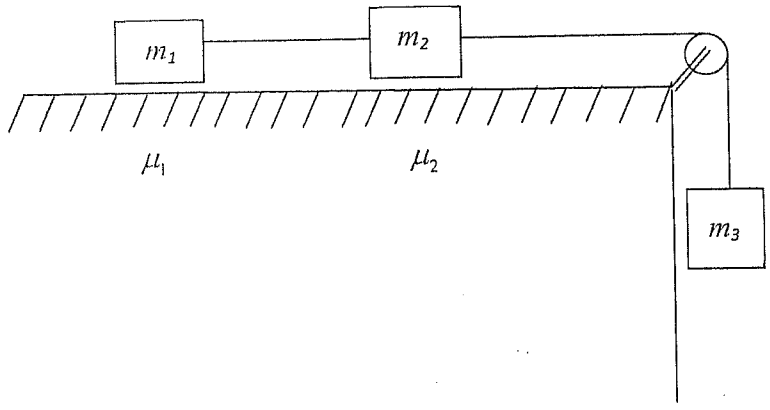
The Open University of Sri Lanka
B.Sc./B.Ed. Degree Programme – Level 05
Open Book Test (OBT) – 2016/2017
Applied Mathematics
APU 3145– Newtonian Mechanics II
Duration :- One Hour

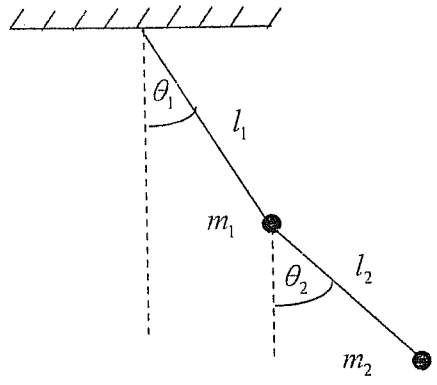
Date :- 08-10-2017 Time :- 10.30 a.m. – 11.30 a.m.

Answer All Questions.

1. Three blocks of masses m_1, m_2 and m_3 are connected as shown in the figure below. The coefficient of friction between the sliding surfaces of the blocks m_1 and m_2 and the plane are μ_1 and μ_2 respectively. Find the acceleration of the masses and tension in the strings using D'Alembert's principle.



2. The double pendulum consists of two bobs of masses m_1 and m_2 at ends of two weightless rods of lengths l_1 and l_2 and one of them is fixed to a rigid support as shown in figure.



(a) Show that the kinetic energy is given by

$$T = \frac{1}{2}(m_1 + m_2) l_1^2 \dot{\theta}_1^2 + \frac{1}{2} m_2 l_2^2 \dot{\theta}_2^2 + m_2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \cos(\theta_1 - \theta_2)$$

(b) Show that the potential energy is given by

$$V = -(m_1 + m_2) g l_1 \cos\theta_1 - m_2 g l_2 \cos\theta_2$$

(c) Hence obtain the Lagrangian of the system.

(d) Show that the Lagrange's equations of motion can be written as

$$(m_1 + m_2) l_1^2 \ddot{\theta}_1 + m_2 l_1 l_2 \ddot{\theta}_2 \cos(\theta_2 - \theta_1) - m_2 l_1 l_2 \dot{\theta}_2^2 \sin(\theta_2 - \theta_1) + (m_1 + m_2) g l_1 \sin\theta_1 = 0 \text{ and}$$

$$m_2 l_2^2 \ddot{\theta}_2 + m_2 l_1 l_2 \ddot{\theta}_1 \cos(\theta_2 - \theta_1) + m_2 l_1 l_2 \dot{\theta}_1^2 \sin(\theta_2 - \theta_1) + m_2 g l_2 \sin\theta_2 = 0.$$