THE OPEN UNIVERSITY OF SRI LANKA
B.Sc. DEGREE PROGRAMME
PURE MATHEMATICS -LEVEL 05
PUU3244/PUE5244 — Number Theory & Polynomials
NO BOOK TEST-2016/2017

DURATION: ONE AND HALF (1 ½) HOURS

Date:- 04.11.2017 **Time:- 10:30a.m. –12:00noon.**

ANSWER ALL QUESTIONS.

1. (i) Express the three forms of Mathematical Induction.

   (ii) Prove by induction that each of the following statements are true:

   (a) $\sum_{i=0}^{n}(-1)^{i} i^{2}=\frac{1}{2}(-1)^{n} n(n+1)$ for all $n \in \mathbb{N}$

   (b) $\sum_{i=0}^{n} i!<(n+1)!$ for all $n \in \mathbb{N}$

   (iii) If $a, b \in \mathbb{Z}$ and $ab = 1$ then prove that
   $|a| = 1$ and $|b| = 1$
   (Hint: You can use the proposition- ***if*** $\boldsymbol{n} \in \mathbb{N}$ ***then*** $\boldsymbol{n} \geq \mathbf{1}$ – without proof.)

   (iv) Prove that $2^{2n} + 1$ is divisible by 5 when n is an odd natural number.

2. (i) Compute the greatest common divisor d of (966, 686, 371) and express it in the form d=966a+686b+371c. where $a,\ b,\ c \in \mathbb{Z}$.

   (ii) If $a \equiv b$ (modm) and $c \equiv d$ (modm) prove each of the following:

   (a) $a + c \equiv b + d \pmod{m}$

   (b) $a - c \equiv b - d \pmod{m}$

   (c) $ac \equiv bd \pmod{m}$

(iii) State Eisentein's irreducibility criteria.

Determine whether the polynomial $f(x) = 2x^{10} - 25x^3 + 10x - 30$ is irreducible over $\mathbb{Q}[x]$.

(iv) Find all irreducible polynomials of degree 2 in $\mathbb{Z}_3[x]$

(03). (i) Find all rational roots of the polynomial $12x^4 - 43x^3 + 53x^2 - 38x + 16$ in $\mathbb{Q}$.

(Hint :Let $f(x) = \sum_{i=0}^{n} a_i x^i \in \mathbb{Z}[x]$ and $n \geq 1$. If $\alpha \in \mathbb{Q}$ is a zero of $f(x)$ and $\alpha = \frac{r}{s}$ with $(r, s) = 1$, then $r \mid a_0$ and $s \mid a_n$.)

(ii) If $f(x) = x^3 + x + 2$ and $g(x) = x^2 + 2$ are polynomials over $\mathbb{Z}_3[x]$. Find the greatest common divisor d of $f(x)$, $g(x)$ and express it in the form $d = fu + gv$ with $u, v \in \mathbb{Z}_3[x]$.