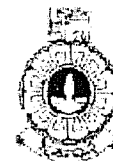


THE OPEN UNIVERSITY OF SRI LANKA
 B.Sc. DEGREE PROGRAMME
 PURE MATHEMATICS -LEVEL 05
 PUU3244/PUE5244 — Number Theory & Polynomials
 NO BOOK TEST-2016/2017



DURATION: ONE AND HALF (1 ½) HOURS

Date:- 04.11.2017

Time:- 10:30a.m. -12:00noon.

ANSWER ALL QUESTIONS.

1. (i) Express the three forms of Mathematical Induction.

(ii) Prove by induction that each of the following statements are true:

$$(a) \sum_{i=0}^n (-1)^i i^2 = \frac{1}{2} (-1)^n n(n+1) \quad \text{for all } n \in \mathbb{N}$$

$$(b) \sum_{i=0}^n i! < (n+1)! \quad \text{for all } n \in \mathbb{N}$$

(iii) If $a, b \in \mathbb{Z}$ and $ab = 1$ then prove that

$$|a| = 1 \text{ and } |b| = 1$$

(Hint: You can use the proposition- *if* $n \in \mathbb{N}$ *then* $n \geq 1$ – without proof.)

(iv) Prove that $2^{2n} + 1$ is divisible by 5 when n is an odd natural number.

2. (i) Compute the greatest common divisor d of (966, 686, 371) and express it in the form $d=966a+686b+371c$. where $a, b, c \in \mathbb{Z}$.

(ii) If $a \equiv b \pmod{m}$ and $c \equiv d \pmod{m}$ prove each of the following:

$$(a) \quad a + c \equiv b + d \pmod{m}$$

$$(b) \quad a - c \equiv b - d \pmod{m}$$

$$(c) \quad ac \equiv bd \pmod{m}$$

(iii) State Eisenstein's irreducibility criteria.

Determine whether the polynomial $f(x) = 2x^{10} - 25x^3 + 10x - 30$ is irreducible over $\mathbb{Q}[x]$.

(iv) Find all irreducible polynomials of degree 2 in $\mathbb{Z}_3[x]$

(03). (i) Find all rational roots of the polynomial $12x^4 - 43x^3 + 53x^2 - 38x + 16$ in \mathbb{Q} .

(Hint :Let $f(x) = \sum_{i=0}^n a_i x^i \in \mathbb{Z}[x]$ and $n \geq 1$. If $\alpha \in \mathbb{Q}$ is a zero of $f(x)$ and $\alpha = \frac{r}{s}$ with $(r, s) = 1$, then $r \mid a_0$ and $s \mid a_n$.)

(ii) If $f(x) = x^3 + x + 2$ and $g(x) = x^2 + 2$ are polynomials over $\mathbb{Z}_3[x]$. Find the greatest common divisor d of $f(x)$, $g(x)$ and express it in the form $d = fu + gv$ with $u, v \in \mathbb{Z}_3[x]$.