THE OPEN UNIVERSITY OF SRI LANKA
B.Sc. DEGREE PROGRAMME
PURE MATHEMATICS -LEVEL 05
PUU3244/PUE5244 — Number Theory & Polynomials
OPEN BOOK TEST-2016/2017

DURATION: ONE AND HALF (1 ½) HOURS

Date: 01.10. 2017 **Time:** 10:30a.m. – 12.00noon

ANSWER ALL QUESTIONS.

1. (i) Which of the following are inductive sets? Justify your answer.

   (a) $[1,\infty)$ (b) $(-1,\infty)$ (c) $(-\infty,10^8)$ (d) $(1,\infty)$

   (ii) Using Mathematical Induction prove that $n! \geq 2^{n-1}$ for all $n \in \mathbb{N}$. Deduce that $\sum_{k=1}^{n} \frac{1}{k!} \leq 2 - \frac{1}{2^{n-1}}$. Hence show that $e \leq 3$; where e is the base of natural logarithm.

   (iii) Show that $(p+q)^n - p^n - q^n$ is divisible by pq where p, q and n are positive integers.

2 (i) Find the greatest common divisor g of 12378 and 3054, and hence find integers x and y such that $12378x + 3054y = g$.

   (ii) Find the least common multiple of 1479 and 272.

   (iii) Let $R = \{a + ib\sqrt{5} : a, b \in \mathbb{Z}\}$. Let $f(x) = 6x^2 + 2\sqrt{5}ix - 1$ and $g(x) = (1 + i\sqrt{5})x - 1$ are in R[x].

   (a) Show that the units in R are ± 1.

   (b) Show that g is a divisor of f.

   (c) Show further that f and g do not have a greatest common divisor.

3. (i) Let $f(x) = x^5 - 2x^4 + 3x - 5$ and $g(x) = 2x + 1$ in $\mathbb{Z}_{11}[x]$. Is $g \mid f$ in $\mathbb{Z}_{11}[x]$? Justify your answer.

(ii) Factorize the polynomial $f(x) = 2x^3 + 3x^2 - 7x - 5$ as a product of linear factors in $\mathbb{Z}_{11}[x]$.

(ii) If F is a field and $a \neq 0$ is a zero of $f(x) = a_0 + a_1x + \cdots + a_nx^n$ in $F[x]$, show that $1/a$ is a zero of $a_n + a_{n-1}x + \cdots + a_0x^n$.