The Open University of Sri Lanka
Department of Mathematics
B.Sc/ B.Ed Degree Programme
Open Book Test (OBT) - 2016/2017
Applied Mathematics– Level 05
APU3244 – Graph Theory

DURATION: - ONE AND HALF HOURS

Date: - 24 – 09– 2017 **Time: - 10.30 a.m. – 12.00 noon**

ANSWER ALL QUESTIONS. THE TOTAL MAXIMUM MARK ATTAINABLE IS 300 AND THE FINAL MARK WILL BE CONVERTED TO 100%.

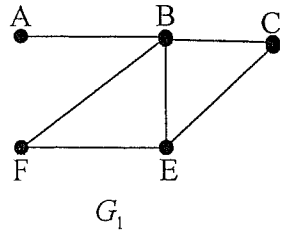
01. (a) Write down five properties of the *Peterson graph* and find, [20 Marks]

(i) A *Hamiltonian path*, [15 Marks]

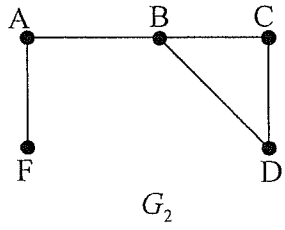
(ii) A *cycle* of length 9 from the Peterson graph. [15 Marks]

Without drawing, find the number of edges that the *complement of the Peterson* graph can have? Justify your answer. [20 Marks]

(b) Construct the *union* of G_1 and G_2, where



G_1



G_2

[15 Marks]

(c) By drawing a graph as an illustrative example, show that $\overline{K_{r,s}} = K_r \cup K_s$. [15 Marks]

02. (a) Determine whether the following graphs are *color critical* or not.

Justify your answer.

(i) C_5 (ii) K_5 (iii) $K_{3,3}$ (iv) $K_{1,n}$ [4×20 Marks]

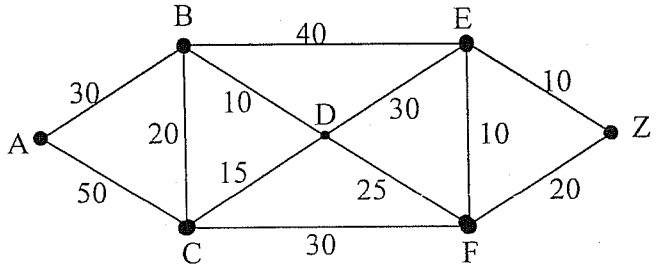
Which of them are *critical planar*? Give reasons for your answer. [25 Marks]

(b) How many different channels are needed for six stations located at the distances shown in the table, if two stations cannot use the same channel when they are within 170 miles of each other?

Station	1	2	3	4	5	6
1	--	85	175	200	50	100
2	85	--	125	175	100	160
3	175	125	--	100	200	250
4	200	175	100	--	210	220
5	50	100	200	210	--	100
6	100	160	250	220	100	--

[25 Marks]

03. (a) Use *Prim's algorithm* to find the minimum weighted spanning tree for the following graph, by starting with the vertex A.



[20 Marks]

By deriving an algorithm similar to Prim's algorithm, construct a *maximum weighted spanning tree* of the above graph. Write down the order of selection of edges in each of those algorithms. [20 Marks]

(b) Find a route, from the city *A*, that visits each of the cities *B*, *C*, *D* and *E* exactly once and return to the starting point with the least total bus fare. The following table gives the least bus fare available for a journey between two cities.

City	*A*	*B*	*C*	*D*	*E*
A	-	110	100	160	50
B	110	-	90	140	150
C	100	90	-	60	130
D	160	140	60	-	120
E	50	150	130	120	-

[30 Marks]