



The Open University of Sri Lanka
 B.Sc. Degree Programme, Level – 04
 Final Examination – 2017/2018
 PYU 2165 – Mathematical Methods for Physics
 Duration: 2 hours

Date: 16th September 2018

Time: 1.30 p.m. to 3.30 p.m.

Answer any four (4) questions

Non-programmable calculators are allowed.

1.

a.

Alice wants to leave a secret message to Bob on a public notice board. She uses cryptography to hide its meaning from others. Her Message is “Physics.” She first encodes it to numbers using the pattern A = 1, B = 2 ... Result is shown in the matrix A. To encrypt it, she uses her encryption matrix B. Encryption is done using a matrices multiplication. Result is matrix C.

$$A = \begin{bmatrix} 16 & 8 \\ 25 & 19 \\ 9 & 3 \\ 19 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 4 & 3 \\ 0 & 3 \end{bmatrix} \quad C = AB$$

i. Calculate C.

ii. Once Bob sees the message, he decrypts it using the decryption matrix D,

$$D = \begin{bmatrix} 1/4 & -1/4 \\ 0 & 1/3 \end{bmatrix} \quad E = CD$$

iii. Calculate the matrix E.

iv. Calculate (BD). What is the special name given to the resultant matrix?

b.

Find the Eigen values of the following matrix

$$X = \begin{bmatrix} -1 & 3 \\ 3 & -9 \end{bmatrix}$$

2.

a. Total work done by moving an object by using a force, $F(x)$, from point a to point b is given by

$$W = \int_a^b F(x) \cdot dx \quad - \text{eqn 1}$$

If $F(x) = x^2$ calculate the total work done by moving it from $x = 10$ to $x = 15$

b.

The elevator in S & T building had stopped between the first and second floors due to a malfunction. The elevator car by itself weighs 500 kg and there are two students weighing 60 kg each inside. To rescue the students, the elevator car must be manually lifted 3 m by winding the cable onto a pulley at the top of the building. ($g = 10 \text{ ms}^{-2}$)

i. Assume cable doesn't have a weight, find $f(x)$, a and b.

ii. Using eqn 1, calculate the total work done.

c.

Assume cable weights 2 kg per meter. At the moment elevator stopped, the length of the cable connecting the elevator and the pulley was 20m.

i. Obtain an expression for the weight felt at the pulley, when elevator had being pulled x meters from the stopped location.

ii. Obtain an expression for the work required to move the elevator dx distance from x position.

iii. Calculate the total work required to rescue.

3.

a.

A small particle is floating on a ripple tank. Its vertical displacement is given by y . A student assumes its acceleration is proportional to the displacement but opposite in direction. He writes following equation.

$$\frac{d^2y}{dt^2} = -py; \text{ where } p > 0 \text{ and is a constant}$$

i. Write the characteristic equation for above O.D.E.

- ii. Find the general solution to above O.D.E.
- iii. He also notices at the beginning displacement is zero and maximum height of A is reached when time is $\frac{\pi}{2}$.

$$y(0) = 0; y\left(\frac{\pi}{2}\right) = A$$

Using above initial conditions find the actual solution.

- b. Later student notices movement is reduced with time and creates a new equation

$$y'' - 8y' + 17y = 0$$

Find the general solution for above equation

- 4.
 - a. Explain a test to determine the convergence of a geometric series?

b. Following is a simplified story told by Greek philosopher Zeno. A rabbit and a tortoise decides to have a race. Rabbit can run 10ms^{-1} and tortoise can run 5ms^{-1} . Since tortoise is slow, it is given a 100 m lead at the beginning of the race.

According to Zeno, it will take 10 second for rabbit to come to the place where tortoise had started the race. By that 10 seconds, tortoise had moved 50 m. When rabbit covers that 50m tortoise had moved 25m...and so forth. For the first few steps, the distance between rabbit and tortoise can be tabulated as follows

Step number	Distance between	Time taken
0	100	0
1	50	10
2	25	5
...

- i. Calculate the distance between rabbit and tortoise in step number 10.
- ii. How many steps are required for distance between rabbit and tortoise to be zero.
- iii. Zeno said 'rabbit can never overtake the tortoise'. **Using your knowledge in sequences and series**, calculate if it is possible for rabbit to overtake the tortoise. If so how long it will take from the start of the race?

5.

a.

- i. What is the difference between a scalar and a scalar field?
- ii. Given an example for a scalar field.

b.

Electric potential (V) inside a chamber is described by the following scalar function.

$$V = x^3 - 2xy + zx + \sin(x)$$

- i. Electric field (\vec{E}) is defined as the gradient of the potential, i.e. $\vec{E} = -\nabla V$. Obtain an expression for the electric field inside the chamber.
- ii. Find the divergence of the above electric field.
- iii. Find the curl of the above electric field.

6.

a.

Magnitude of the electric field, created at the center of an electric dipole is given by the following formula.

$$E_r = \frac{2q}{4\pi\epsilon} \left[\frac{1}{r^2} - \frac{r}{(r^2 + d^2)^{\frac{3}{2}}} \right]$$

- i. Show that the above expression can be re-written as

$$E_r = \frac{2q}{4\pi\epsilon r^2} \left[1 - \left(1 + \frac{d^2}{r^2} \right)^{-\frac{3}{2}} \right]$$

- ii. Provided $\frac{d^2}{r^2} \ll 1$, show

$$E_r \approx \frac{3qd^2}{4\pi\epsilon r^4}$$

Hint : Binomial expansion for any real value of P, if $|x| < 1$ is given by

$$(1+x)^p = \sum_{r=0}^{\infty} \frac{p(p-1)\dots[p-(r-1)]}{r!} x^r$$

b.

Interference of two audio sources creates beats. S_A and S_B are two sound sources with following amplitudes.

$$S_A = P \sin(\omega_1 t)$$

$$S_B = P \sin(\omega_2 t)$$

- i. Show that superposition of above two waves, (*i. e.* $S_A + S_B$), can be described as a product of sine and cosine functions.
- ii. If ω_1 is 7 rads^{-1} and ω_2 is 5 rads^{-1} what is the lowest and highest frequencies you can get from superposition.

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