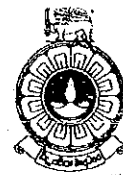


THE OPEN UNIVERSITY OF SRI LANKA
 B.Sc./B.Ed DEGREE PROGRAMME – LEVEL 04
 FINAL EXAMINATION -2017/2018
 PURE MATHEMATICS
 PEU4302/PEE4302/PUU2142 – Linear Algebra
 DURATION: Two (02) Hours



Date: 23.09.2018

Time: 1.30p.m.-3.30p.m.

Answer FOUR questions only.

1. (a) Define each of the following:

- (i) Equivalent matrices,
- (ii) Echelon form,
- (iii) Orthogonal matrix,
- (iv) Minor of a matrix.

(b) Show that $x = 0$ is one of the roots of the following equation:

$$\begin{vmatrix} 0 & x-a & x-b \\ x+a & 0 & x-c \\ x+b & x+c & 0 \end{vmatrix} = 0.$$

(c) If $A = \begin{pmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{pmatrix}$, then show that $A^3 = A^{-1}$.

(d) Determine whether the following pair of matrices are equivalent or not. Justify your answer.

$$\begin{pmatrix} 2 & -1 & 3 & 4 \\ 0 & 3 & 4 & 1 \\ 2 & 3 & 7 & 5 \\ 2 & 5 & 11 & 5 \end{pmatrix}, \begin{pmatrix} 1 & 0 & -5 & 6 \\ 3 & -2 & 1 & 2 \\ 5 & -2 & -9 & 14 \\ 4 & -2 & -4 & 8 \end{pmatrix}.$$

2. (a) Show that $\begin{pmatrix} \cos \phi & 0 & \sin \phi \\ \sin \theta \sin \phi & \cos \theta & -\sin \theta \cos \phi \\ -\cos \theta \sin \phi & \sin \theta & \cos \theta \cos \phi \end{pmatrix}$ is an orthogonal matrix.

(b) Find non-singular matrices P and Q such that PAQ is normal, where

$$A = \begin{pmatrix} 1 & 2 & 3 & -2 \\ 2 & -2 & 1 & 3 \\ 3 & 0 & 4 & 1 \end{pmatrix}.$$

(c) Find the rank of the following matrix:

$$\begin{pmatrix} 1 & 3 & 2 & 5 & 1 \\ 2 & 2 & -1 & 6 & 3 \\ 1 & 1 & 2 & 3 & -1 \\ 0 & 2 & 5 & 2 & -3 \end{pmatrix}.$$

3. (a) For which values of λ and μ , the following system of equations will have

- (i) a unique solution,
- (ii) no solution,
- (iii) infinitely many solutions.

$$2x + 3y + 5z = 9$$

$$7x + 3y - 2z = 8$$

$$2x + 3y + \lambda z = \mu.$$

(b) Let $A = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$.

Find the characteristic equation of A and verify that it is satisfied by A . Hence obtain A^{-1} .

Express $A^6 - 6A^5 + 9A^4 - 2A^3 - 12A^2 + 23A - 9I$ as a linear polynomial in A .

(c) Find the eigen values and the corresponding eigen vectors of the matrix

$$\begin{pmatrix} 6 & -3 & 3 \\ -3 & 6 & -3 \\ 3 & -3 & 6 \end{pmatrix}.$$

4. (a) Transform the following quadratic form to canonical form by an orthogonal transformation:

$$x_1^2 + 5x_2^2 + x_3^2 + 2x_1x_2 + 2x_2x_3 + 6x_3x_1.$$

- (b) Determine the nature, index and the signature of the above quadratic form.

5. (a) If $A = \begin{pmatrix} 0 & -\tan \frac{\alpha}{2} \\ \tan \frac{\alpha}{2} & 0 \end{pmatrix}$ and I is the 2×2 unit matrix, then

$$\text{show that } I + A = (I - A) \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix}.$$

- (b) Solve the following system of linear equations by LU-decomposition:

$$x_1 + x_2 + x_3 + x_4 = 0$$

$$x_1 + x_2 + x_3 - x_4 = 4$$

$$x_1 + x_2 - x_3 + x_4 = -4$$

$$x_1 - x_2 + x_3 + x_4 = 2.$$

6. (a) Find the Hermitian form of

$$A = \begin{bmatrix} 3 & 2-i \\ 2+i & 4 \end{bmatrix} \text{ with } X = \begin{bmatrix} 1+i \\ 2i \end{bmatrix}$$

- (b) Find the Skew-Hermitian form for each of the following matrices:

(i) $A = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}$ with $X = \begin{pmatrix} 1 \\ i \end{pmatrix}$,

(ii) $A = \begin{pmatrix} 2i & 4 \\ -4 & 0 \end{pmatrix}$ with $X = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$.