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**THE OPEN UNIVERSITY OF SRI LANKA**

**B.Sc./B.Ed DEGREE PROGRAMME – LEVEL 04**

**FINAL EXAMINATION -2017/2018**

**PURE MATHEMATICS**

**PEU4302/PEE4302/PUU2142 – Linear Algebra**

**DURATION: Two (02) Hours**

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**Date: 23.09.2018** **Time:1.30p.m.-3.30p.m.**

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**Answer FOUR questions only.**

1. (a) Define each of the following:

   (i) Equivalent matrices,
   (ii) Echelon form,
   (iii) Orthogonal matrix,
   (iv) Minor of a matrix .

   (b) Show that $x=0$ is one of the roots of the following equation:

$$\begin{vmatrix} 0 & x-a & x-b \\ x+a & 0 & x-c \\ x+b & x+c & 0 \end{vmatrix} = 0.$$

   (c) If $A=\begin{pmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{pmatrix}$, then show that $A^3 = A^{-1}$.

   (d) Determine whether the following pair of matrices are equivalent or not. Justify your answer.

$$\begin{pmatrix} 2 & -1 & 3 & 4 \\ 0 & 3 & 4 & 1 \\ 2 & 3 & 7 & 5 \\ 2 & 5 & 11 & 5 \end{pmatrix}, \begin{pmatrix} 1 & 0 & -5 & 6 \\ 3 & -2 & 1 & 2 \\ 5 & -2 & -9 & 14 \\ 4 & -2 & -4 & 8 \end{pmatrix}.$$

2. (a) Show that $\begin{pmatrix} \cos\phi & 0 & \sin\phi \\ \sin\theta\sin\phi & \cos\theta & -\sin\theta\cos\phi \\ -\cos\theta\sin\phi & \sin\theta & \cos\theta\cos\phi \end{pmatrix}$ is an orthogonal matrix.

(b) Find non- singular matrices $P$ and $Q$ such that $PAQ$ is normal, where

$$A = \begin{pmatrix} 1 & 2 & 3 & -2 \\ 2 & -2 & 1 & 3 \\ 3 & 0 & 4 & 1 \end{pmatrix}.$$

(c) Find the rank of the following matrix:

$$\begin{pmatrix} 1 & 3 & 2 & 5 & 1 \\ 2 & 2 & -1 & 6 & 3 \\ 1 & 1 & 2 & 3 & -1 \\ 0 & 2 & 5 & 2 & -3 \end{pmatrix}.$$

3. (a) For which values of $\lambda$ and $\mu$, the following system of equations will have

(i) a unique solution,
(ii) no solution,
(iii) infinitely many solutions.

$$2x+3y+5z=9$$
$$7x+3y-2z=8$$
$$2x+3y+\lambda z=\mu.$$

(b) Let $A = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}.$

Find the characteristic equation of $A$ and verify that it is satisfied by $A$. Hence obtain $A^{-1}$.
Express $A^6 - 6A^5 + 9A^4 - 2A^3 - 12A^2 + 23A - 9I$ as a linear polynomial in $A$.

(c) Find the eigen values and the corresponding eigen vectors of the matrix

$$\begin{pmatrix} 6 & -3 & 3 \\ -3 & 6 & -3 \\ 3 & -3 & 6 \end{pmatrix}.$$

4. (a) Transform the following quadratic form to canonical form by an orthogonal transformation:

$$x_1^2 + 5x_2^2 + x_3^2 + 2x_1x_2 + 2x_2x_3 + 6x_3x_1 .$$

(b) Determine the nature, index and the signature of the above quadratic form.

5. (a) If $A = \begin{pmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{pmatrix}$ and $I$ is the $2\times 2$ unit matrix, then show that $I + A = (I - A)\begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$.

(b) Solve the following system of linear equations by LU-decomposition:

$$\begin{aligned} x_1 + x_2 + x_3 + x_4 &= 0 \\ x_1 + x_2 + x_3 - x_4 &= 4 \\ x_1 + x_2 - x_3 + x_4 &= -4 \\ x_1 - x_2 + x_3 + x_4 &= 2. \end{aligned}$$

6. (a) Find the Hermitian form of

$$A = \begin{bmatrix} 3 & 2-i \\ 2+i & 4 \end{bmatrix} \text{with } X = \begin{bmatrix} 1+i \\ 2i \end{bmatrix}$$

(b) Find the Skew- Hermitian form for each of the following matrices:

(i) $A = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}$ with $X = \begin{pmatrix} 1 \\ i \end{pmatrix}$,

(ii) $A = \begin{pmatrix} 2i & 4 \\ -4 & 0 \end{pmatrix}$ with $X = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$.