THE OPEN UNIVERSITY OF SRI LANKA
B.SC. DEGREE PROGRAMME: LEVEL 05
DEPARTMENT OF COMPUTER SCIENCE
CSU 5304/CPU3140 – MATHEMATICS FOR COMPUTING
Final Examination - 2017/18



DURATION: Two Hours Only (2 Hours)

Date: 17th September 2018

Time: 09.30am-11.30am

Answer four Questions only

- 1) (i) Describe the principle of Mathematical Induction for a statement P(n) where $n \in \mathbb{N}$.
 - (ii) Prove the following identity, for any positive integer k using the above principle.

$$1+3+3^2+\ldots+3^{k-1}=\frac{3^k-1}{2}$$

- (iii) Use Mathematical Induction to verify that for any natural number n, $10^{(2n-1)}+1$ is divisible by 11.
- 2) (i) Give the definition of a matrix. What do you mean by the order of a matrix?
 - (ii) Given that $A = \begin{bmatrix} 0 & 1 & -1 \\ 4 & 3 & -4 \\ 3 & 3 & -4 \end{bmatrix}$ verify that $A^2 = I$, where I is the Identity matrix of the same size. Hence find A^{-1}
 - (iii) If B is a square matrix of any order such that $B^2=B$ Show that $(I+B)^3=7B+I$ where I is the identity matrix of the same order of B.
 - (iv) Find the determinant of $\begin{bmatrix} 2 & -3 & 5 \\ -3 & 6 & 2 \\ 1 & -2 & 5 \end{bmatrix}$ expanding along the 1st column.
- 3) (i) A and B are two sets. Draw appropriate <u>Venn Diagrams</u> for each of the following sets.

(a)
$$(A \cup B)^1$$
, (b) $A^1 \cap B^1$, (c) $(A \cap B)^1$ (d) $A^1 \cup B^1$

- (ii) Write the following sets in roster form.
 - (a) $A=\{x:x \text{ is an integer and } -3 \le x \le 7\}$
 - (b) B= {The set of all letters in the word **BETTER**}
 - (c) C={x:x is a natural number less than 6}
- (iii) Are the following pairs of sets equal? Justify your answer.
 - (a) A= $\{2,3\}$ and B= $\{x:x \text{ is a solution of } x^2+5x+6=0\}$
 - (b) C={x:x is a letter in the word **FOLLOW**} and D={y: y is a letter in the word **WOLF**}
- (iv) In a group of 70 students, 37 like coffee, 52 like tea and each student likes at least one of the two drinks. How many students like both tea and coffee?
- (v) Using set identities, prove that

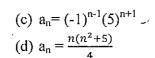
$$A \cap (B - C) = (A \cap B) - (A \cap C)$$

- 4) (i) Give the definitions of the following
 - (a) Function
 - (b) One to One Function
 - (c) On-to Function
 - (ii) f and g are two functions such that $f(x)=3x^2-5$ and $g(x)=\frac{x}{3x^2+1} \ \forall \ x \in \mathbb{R}$ Find the composition function of **gof** and the inverse of **f**.
 - (iii) Let h: $\mathbb{R} \to \mathbb{R}$ be defined by

$$h(x) = \begin{cases} 2x; x > 3\\ x^2; 1 < x \le 3\\ 3x; x \le 1 \end{cases}$$

Find the value of h(-1) + h(2) + h(4)

- 5) (i) Write the first five terms of each of the sequences whose nth term is given below,
 - (a) $a_n = n(n+2)$
 - (b) $a_n = \frac{n}{n+2}$



- (ii) Given that $a_1 = 3$ and $a_n = 3a_{n-1} \ \forall \ n > 1$ find the required sequence and the Corresponding series.
- (iii) (a) Find the sum of the Arithmetic series of odd integers from 1 to 2001.
 - (b) The sum of the first n terms of an Arithmetic series is (pn+qn²) where p and q are constants. Find the common difference.
 - (c) A person starts repaying a loan as Rs100/= as the first installment. If he increases the installment by Rs 5/= every month, what amount will he pay in the 30th installment?
- (iv) (a) In a Geometric Progression terms are given as $\frac{5}{2}$, $\frac{5}{4}$, $\frac{5}{8}$,.... Find the 20th term.
 - (d) Find the sum of the first 20 terms of the above progression.
- 6) (i)Suppose there are two statements p and q such that $p\rightarrow q$.

Write the converse and the contrapositive of $p\rightarrow q$.

Give the converse and the contrapositive of the following sentences.

- (a) We will play the game, if it is sunny.
- (b) If it rains today, I will stay at home.
- (c) I drive if it is too dangerous or too far to bike.
- (ii) (a) What is a "Propositional Statement"?
 - (b)Construct a truth table for $p \to \neg q \land (q \lor p)$ Use the above truth table to decide whether the proposition is a Tautology, Contradiction or a Contingency.
- (iii) Using logical equivalences or laws, prove the following algebraically.

$$\neg qV [\neg(pV\neg p) \land r] \rightarrow s \equiv sVq$$

- (iv) Find the truth values of the following formulas in Predicate Logic.
 - (a) $\forall x P(x) \leftrightarrow \neg \exists x \neg P(x)$
 - (b) $\exists x P(x) \leftrightarrow \neg \forall x P(x)$
 - (c) $\forall x \exists y P(x,y) \leftrightarrow \exists y \forall x P(x,y)$
- (v) Translate each of the following statements into logical expressions using predicates, quantifiers and logical connectives.

Predicates are given bellow.

C(x): x is a CSU 260 student.

L(x): x loves music.

The universe of discourse for the variable x is all students.

- (a) Every student loves music.
- (b) No students loves music.
- (c) Some students love music.
- (d) Every CSU260 student loves music.
- (e) Some CSU260 students love music.

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