THE OPEN UNIVERSITY OF SRI LANKA
B.SC. DEGREE PROGRAMME: LEVEL 05
DEPARTMENT OF COMPUTER SCIENCE
CSU 5304/CPU3140 – MATHEMATICS FOR COMPUTING
Final Examination - 2017/18

DURATION:Two Hours Only (2 Hours)

Date : 17th September 2018 **Time: 09.30am-11.30am**

Answer four Questions only

1) (i) Describe the principle of Mathematical Induction for a statement P(n) where $n \in \mathbb{N}$.

(ii) Prove the following identity, for any positive integer k using the above principle.

$1+3+3^2+\ldots+3^{k-1} = \frac{3^k-1}{2}$

(iii) Use Mathematical Induction to verify that for any natural number n, $10^{(2n-1)}+1$ is divisible by11.

2) (i) Give the definition of a matrix. What do you mean by the order of a matrix?

(ii) Given that $A=\begin{bmatrix} 0 & 1 & -1 \\ 4 & 3 & -4 \\ 3 & 3 & -4 \end{bmatrix}$ verify that $A^2=I$, where I is the Identity matrix of the same size. Hence find A^{-1}

(iii) If B is a square matrix of any order such that $B^2=B$
Show that $(I+B)^3= 7B+I$ where I is the identity matrix of the same order of B.

(iv) Find the determinant of $\begin{bmatrix} 2 & -3 & 5 \\ -3 & 6 & 2 \\ 1 & -2 & 5 \end{bmatrix}$ expanding along the 1st column.

3) (i) A and B are two sets. Draw appropriate <u>Venn Diagrams</u> for each of the following sets.

(a) $(A \cup B)^1$, (b) $A^1 \cap B^1$, (c) $(A \cap B)^1$ (d) $A^1 \cup B^1$

(ii) Write the following sets in roster form.

(a) A={x:x is an integer and -3<x<7}

(b) B= {The set of all letters in the word **BETTER**}

(c) C={x:x is a natural number less than 6}

(iii) Are the following pairs of sets equal? Justify your answer.

(a) A={2,3} and B={x:x is a solution of $x^2+5x+6=0$}
(b) C={x:x is a letter in the word **FOLLOW**} and D={y: y is a letter in the word **WOLF**}

(iv) In a group of 70 students, 37 like coffee, 52 like tea and each student likes at least one of the two drinks. How many students like both tea and coffee?

(v) Using set identities, prove that

$$A \cap (B - C) = (A \cap B) - (A \cap C)$$

4) (i) Give the definitions of the following
(a) Function
(b) One to One Function
(c) On-to Function

(ii) f and g are two functions such that $f(x)=3x^2-5$ and $g(x) = \frac{x}{3x^2+1} \forall x \in \mathbb{R}$

Find the composition function of **g∘f** and the inverse of **f**.

(iii) Let h: $\mathbb{R} \rightarrow \mathbb{R}$ be defined by

$$h(x) = \begin{cases} 2x; x > 3 \\ x^2; 1 < x \leq 3 \\ 3x; x \leq 1 \end{cases}$$

Find the value of h(-1) + h(2) + h(4)

5) (i) Write the first five terms of each of the sequences whose n^{th} term is given below,
(a) $a_n = n(n+2)$
(b) $a_n = \frac{n}{n+2}$

(c) $a_n = (-1)^{n-1}(5)^{n+1}$

(d) $a_n = \frac{n(n^2+5)}{4}$

(ii) Given that $a_1 = 3$ and $a_n = 3a_{n-1} \; \forall \; n > 1$ find the required sequence and the Corresponding series.

(iii) (a) Find the sum of the Arithmetic series of odd integers from 1 to 2001.

(b)The sum of the first n terms of an Arithmetic series is $(pn+qn^2)$ where p and q are constants. Find the common difference.

(c) A person starts repaying a loan as Rs100/= as the first installment. If he increases the installment by Rs 5/= every month, what amount will he pay in the 30^{th} installment?

(iv) (a) In a Geometric Progression terms are given as $\frac{5}{2}, \frac{5}{4}, \frac{5}{8}, \ldots$. Find the 20^{th} term.

(d) Find the sum of the first 20 terms of the above progression.

6) (i)Suppose there are two statements p and q such that $p \rightarrow q$.

Write the converse and the contrapositive of $p \rightarrow q$.

Give the converse and the contrapositive of the following sentences.

(a) We will play the game, if it is sunny.
(b) If it rains today, I will stay at home.
(c) I drive if it is too dangerous or too far to bike.

(ii) (a) What is a **"Propositional Statement"?**

(b)Construct a truth table for $p \rightarrow \neg q \wedge (q \vee p)$
Use the above truth table to decide whether the proposition is a Tautology, Contradiction or a Contingency.

(iii) Using logical equivalences or laws, prove the following algebraically.

$\neg \; q \vee [\neg(p \vee \neg p) \wedge r] \rightarrow s \equiv s \vee q$

(iv) Find the truth values of the following formulas in Predicate Logic.

(a) $\forall x P(x) \leftrightarrow \neg\exists x \neg P(x)$
(b) $\exists x\ P(x) \leftrightarrow \neg\forall x\ P(x)$
(c) $\forall x \exists y\ P(x,y) \leftrightarrow \exists y \forall x\ P(x,y)$

(v) Translate each of the following statements into logical expressions using predicates, quantifiers and logical connectives.

Predicates are given bellow.

C(x): x is a CSU 260 student.

L(x): x loves music.

The universe of discourse for the variable x is all students.

(a) Every student loves music.
(b) No students loves music.
(c) Some students love music.
(d) Every CSU260 student loves music.
(e) Some CSU260 students love music.