THE OPEN UNIVERSITY OF SRI LANKA
B.Sc. /B.Ed. Degree Programme
APPLIED MATHEMATICS-LEVEL 05
ADU5300/APU3141- Linear Programming
FINAL EXAMINATION 2017/2018
Duration: Two Hours

Date: 13.09.2018	**Time: 01.30 p.m- 03.30 p.m**

Answer four questions only

(1) A firm can produce 3 types of cloth, *A, B* and *C* using red, green and blue colours of wool. One unit of type *A* needs 2 meters of red wool and 3 meters of blue wool. One unit of type *B* needs 3 meters of red wool, 2 meters of green wool and 2 meters of blue wool. One unit of type *C* needs 5 meters of green wool and 4 meters of blue wool. The firm has a stock of 8 meters of red, 10 meters of green and 15 meters of blue. It is assumed that the income obtained from one unit of type *A* is Rs.3, *B* is Rs.5 and *C* is Rs.4.

(i) Formulate the above as a Linear Programming model to maximize the total income.
(ii) Using the Simplex method, solve the model in part (i) to determine how much quantity should be produced from each cloth in order to maximize the income for the firm.
(iii) Find the maximum income.

(2) A dairy firm has three plants, say *A*, *B*, *C*, located throughout a province. Daily milk production at each plant is 6, 1 and 10 million liters respectively. Each day the firm must fulfill the needs of its four distribution centres, say *W*, *X*, *Y*, *Z*. Milk requirement at each centre is 7, 5, 3 and 2 million liters respectively. Cost of shipping one million liters of milk from each plant to each distribution centre is given in the following table in hundreds of rupees:

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Distribution Centre / Plant	*W*	*X*	*Y*	*Z*
A	2	3	11	7
B	1	0	6	1
C	5	8	15	9

The dairy firm wishes to determine as to how much should be the shipment from which milk plant to which distribution centre so that the total cost of shipment is minimized.

(i) Formulate a linear programming model to determine the minimum cost of shipment.

(ii) Find the initial solution using Vogel's Approximation method in order to minimize the cost of shipment.

(iii) Find the optimal solution.

(3) The captain of a cricket team has to allot five middle batting positions to five batsmen. The average runs scored by each batsman at these positions are as follows:

		Batting position				
Batsman		*I*	*II*	*III*	*IV*	*V*
	P	40	40	35	25	50
	Q	42	30	16	25	27
	R	50	48	40	60	50
	S	20	19	20	18	25
	T	58	60	59	55	53

(i) Find the assignment of batsmen to positions which would give the maximum number of runs.

(ii) Another batsman 'U' with the following average runs in batting positions as given below is considered to be added to the team:

Batting position	*I*	*II*	*III*	*IV*	*V*
Average runs	45	52	38	50	49

Should he be included to play in the team? If so, who will be replaced by him?

(4) a) Briefly explain the following terms in linear programming:

(i) Feasible solution

(ii) Objective function

(iii)Unbounded solution

(iv)Optimal solution.

b) The objective function and the initial simplex tableau for a profit maximization linear programming problem is given below. Here, X_1 and X_2 represent number of units of product A and B respectively. S_1 and s_2 represent slack variables which cooperates with constraint 1 and 2 respectively. Z represents the total profit:

Maximize $z = 3x_1 + 4x_2$

Basis	X_1	X_2	s_1	s_2	Solution
s_1	2	3	1	0	16
s_2	4	2	0	1	16
-Z	3	4	0	0	0

(i) Write down the two constraints as inequalities with integer coefficients.

(ii) Is the solution feasible? Justify your answer.

(iii) Is the solution optimal? If it is not optimal, find out the optimal solution.

(iv) How many of X_1 and X_2 products are needed to produce to maximize the total profit?

(5) Soft drink manufacturing company produces an orange flavored soft drink by combining orange soda and orange juice. Each ounce of orange soda contains 0.5 oz of sugar and 1 oz of vitamin C. Each ounce of orange juice contains 0.25 oz of sugar and 3 oz of vitamin C. It costs Rs.2 to produce an ounce of orange soda and Rs.3 to produce an ounce of orange juice. Marketing Department has decided that each 10 oz bottle of that soft drink must contain at least 20 oz of vitamin C and at most 4 oz of sugar.

Use a Linear Programming model to determine how manufacturing company can meet Marketing Department's requirements at the minimum cost.

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(6) Consider the following linear programming problem:

$$\text{Maximize } z = y_1 - y_2 + 3y_3,$$

$$\text{Subject to } y_1 + y_2 + y_3 \leq 10,$$

$$2y_1 - y_3 \leq 2,$$

$$2y_1 - 2y_2 + 3y_3 \leq 6,$$

$$y_1, y_2, y_3 \geq 0.$$

(i) Write down the dual linear programme for the above primal problem.

(ii) Solve the dual linear programme obtained in part (i) by using the dual simplex method. Hence, write down the optimal solution to the primal problem.

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