00176

The Open University of Sri Lanka
B.Sc./B.Ed. Degree Programme
Final Examination 2017/2018
Applied Mathematics – Level 04

ADU4301/APU2142– Newtonian Mechanics I

Duration :- Two Hours

Date :-03.04.2019 **Time:- 01.30p.m.-3.30 p.m.**

Answer Four Questions Only.

1. (i) With the usual notation, show that in intrinsic coordinates the velocity and acceleration components of a particle moving in a plane curve are given by $\underline{v} = \dot{s}\,\underline{t}$ and $\underline{a} = \ddot{s}\,\underline{t} + \frac{\dot{s}^2}{\rho}\,\underline{n}$ respectively.

(ii) A smooth wire in the form of an arch of a cycloid with intrinsic equation: $s = 4a\sin\psi$, $-\frac{\pi}{2} \le \psi \le \frac{\pi}{2}$ is fixed in a vertical plane, the vertex O being the lowest point of the wire where the tangent is horizontal. A bead, of mass m, which can slide freely on the wire, is released from rest at the point where $\psi = \frac{\pi}{6}$.

(a) Find the periodic time of oscillation of the bead.

(b) Show that the normal contact force exerted by the wire on the bead at a point where the tangent makes an angle ψ with the horizontal is:

$$\frac{1}{4}mg\sec\psi(8\cos^2\psi - 3)$$

2. (i) With the usual notation, show that in plane polar coordinates, the velocity and acceleration components of a particle moving in a plane are given by $\underline{v} = \dot{r}\underline{e}_r + r\dot{\theta}\underline{e}_\theta$ and

$\underline{a} = (\ddot{r} - r\dot{\theta}^2)\underline{e}_r + \frac{1}{r}\frac{d(r^2\dot{\theta})}{dt}\underline{e}_\theta$ respectively.

(ii) A satellite *S*, of mass *m*, is orbiting the earth in a plane through the centre *O* of the earth, which may be regarded as stationary. The only force on the satellite has magnitude F and acts along *SO*. At time *t*, *OS* is of length *r* and is rotating about *O* with angular speed $\dot{\theta}$. When $t = 0$, $r = a$ and the satellite is moving with speed *V* in a direction perpendicular to *OS*.

(a) Show that $r^2\dot{\theta} = aV$

(b) Given that $F = \dfrac{2maV^2}{r^2}$, show that $\ddot{r} = \dfrac{aV^2}{r^3}(a - 2r)$.

(c) Hence show that $\dot{r}^2 = \dfrac{V^2}{r^2}(4ar - a^2 - 3r^2)$.

(d) Deduce that the satellite is moving in a direction perpendicular to *OS* when $r = a/3$ also.

3. (i) With the usual notation show that the equation of a central orbit is given by

$$\frac{d^2u}{d\theta^2} + u = \frac{F}{h^2u^2} \quad \text{and} \quad \dot{\theta} = hu^2.$$

(ii) A particle *P* moves in a path with polar equation $r = \dfrac{2a}{2 + \cos\theta}$, coordinates being measured with respect to a pole *O* and initial line *OA*. Given that at any time *t* during the motion $r^2\dot{\theta} = h$ (constant), determine the central force.

4. (i) Establish the formula $\underline{F}(t) = m(t)\dfrac{d\underline{v}}{dt} + \underline{u}\dfrac{dm}{dt}$ for the motion of a particle of varying mass $m(t)$ moving with velocity $\underline{v}$ under a force $\underline{F}(t)$, the matter being emitted at a rate $\dfrac{dm}{dt}$ with velocity $\underline{u}$ relative to the particle.

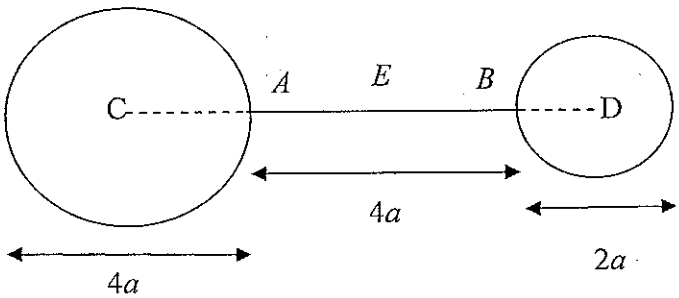
(ii) A particle P, whose initial mass is m_0, is projected vertically upwards from the ground at time $t=0$ with speed g/k, where k is a constant. As the particle moves upwards it gains mass by picking up continously small droplets of moisture from a cloud. The droplets are at rest before they are picked up. At time t the speed of P is v and its mass has increased to $m_0 e^{kt}$. Assuming that, during the motion, the acceleration due to gravity is constant,

(a) show that, while P is moving upwards, $kv+\frac{dv}{dt}=-g$

(b) find, in terms of m_0, the mass of P when it reaches its greatest height above the ground.

5. (a) Show that the moment of inertia of a uniform solid sphere, of mass m and radius a, about a diameter is $\frac{2ma^2}{5}$.

(b) A body consists of two uniform solid spheres and a uniform rod. The uniform rod, AB, has mass $2m$ and length $4a$. The larger sphere has mass $4m$ and diameter $4a$ and is rigidly attached at A. The smaller sphere has mass m and diameter $2a$ and is rigidly attached at B. The centres of the spheres are C and D, and CABD is a straight line, as shown in the diagram.



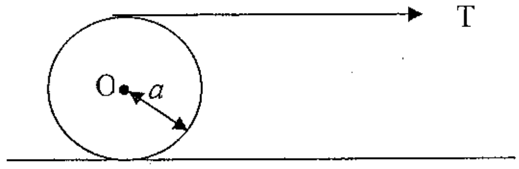
The rod is smoothly pivoted at E, the midpoint of AB. The body is free to rotate about a horizontal axis through E and perpendicular to AB. Initially the body is at rest with AB horizontal.

(i) Show that the moment of inertia of the smaller sphere about the axis through E is $\frac{47ma^2}{5}$.

(ii) Find the moment of inertia of the body about the axis through E.

(iii) Find the maximum angular velocity of the body after it is released from rest.

6. A uniform solid cylinder of mass M kg and radius a m has a string attached to a point of its surface and wound several times around the cylinder. The cylinder rests with its curved surface on a rough horizontal plane and the string is pulled in a direction parallel to the plane and perpendicular to the axis of the cylinder as shown in the diagram. Assume that the motion is confined to a vertical plane through O, the centre of the circuar cross-section.



If the cylinder rolls without slipping on the plane, write down a set of equations sufficient to determine μ, the coefficient of friction between the cylinder and the horizontal plane. If the cylinder starts to roll when $M = 8kg$, $a = 0.25m$ and the tension $T = 30N$ find minimum value of μ.