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The Open University of Sri Lanka
B.Sc. / B.Ed. Degree Programme
Final Examination -2017/2018
Applied Mathematics – Level 05
APU3240/APE5240 — Numerical Methods

Duration: Three Hours

Date: 18. 09. 2018 **Time: 01.30 p.m. – 04.30 p.m.**

Answer Five Questions Only.

1. (a) Derive Newton- Raphson formula for solving the non-linear equation $f(x)=0$.

   (b) Show that Newton- Raphson method has quadratic convergence.

   (c) Find the root of the equation $x^3-x-1=0$, lying in the interval [1, 2], taking x_0 as 1.5, correct to four decimal places, using method of Newton- Raphson.

2. (a) Prove that

   (i) $E=\Delta+1$, (ii) $\Delta=E^{1/2}\delta$,
   (iii) $\Delta^2=\delta^2+E^{1/2}\delta^3$, (iv) $\Delta^3=E^{1/2}\delta^3+\delta^4+E^{1/2}\delta^5$,

   where Δ, δ and E are the forward difference, the central difference and the shift operators respectively.

   (b) Derive Gregory- Newton forward interpolation formula.

   (c) Hence, interpolate $f(2.5)$, given that $f(x)$ passes through the points (2, 1), (3, 2), (4, 6), (5, 24) and (6, 120).

3. (a) (i) Derive Newton's general interpolation formula with divided differences.

   (ii) Hence, find the fourth degree polynomial that passes through the points

   (1.0, 0.7651977), (1.3, 0.6200860), (1.6, 0.4554022), (1.9, 0.2818186) and

   (2.2, 0.1103623).

   (b) (i) Derive Lagrange's interpolation formula.

(ii) Find the equation of the parabola passing through the points (0, 1), (1, 3), and (3, 55) using Lagrange's interpolation formula.

4. (a) Derive the Simpson's One –Third Rule.

(b) If the interval $[a, b]$ is divided into $2n$ sub intervals and corresponding ordinates are denoted by $y_0,\ y_2 \ldots\ldots y_{2n}$ then show that the error in Simpson's One –Third rule is given by $|E| < \frac{(b-a)h^4}{180} M$, where M is the numerically greater value of $y_0^{iv},\ y_2^{iv} \ldots\ldots y_{2n-2}^{iv}$.

(c) A car completes a lap of a race track in 84 seconds. The speed of the car at each 6-second interval is determined by using a radar gun and is given from the beginning of the lap, in meters/second, by the entries in the following table. Find the length of the lap.

Time	0	6	12	18	24	30	36	42	48	54	60	66	72	78	84
Speed	41	45	49	52	49	44	40	36	33	28	26	30	35	39	41

5. (a) Applying Taylor series method of fourth order for the differential equation

$\frac{dy}{dx} = x + y$ subject to the initial condition $y(1) = 0$, evaluate $y(1.1)$ and $y(1.2)$.

(b) Applying Taylor series method of third order for the system of differential equations $\frac{dx}{dt} = x + y + t$ and $\frac{dy}{dt} = 2x - t$ subject to the initial conditions $x = 0,\ y = 1$ at $t = 1$, show that $x(t) = 2t + t^2 + \frac{5}{6}t^3$ and $y(t) = 1 - t + \frac{3}{2}t^2 + \frac{2}{3}t^3$.

6. (a) (i) Derive formula for Picard's method to solve $\frac{dy}{dx} = f(x, y)$ subject to the initial condition $y(x_0) = y_0$.

(ii) Using Picard's method, find the first-three successive approximations to solve $\frac{dy}{dx} = e^x + y^2$ with the initial condition $y(0) = 0$.

(b) Applying Modified Euler method for $\frac{dy}{dx} = x + y$ subject to the initial condition $y(0) = 0$, taking $h = 0.2$, evaluate $y(0.2)$ and $y(0.4)$.

7. (a) State fourth order Runge-Kutta algorithm to solve $\frac{dy}{dx} = f(x, y)$ subject to the initial condition $y(x_0) = y_0$.

(b) Solve $\frac{dy}{dx} = \frac{1 - xy}{x^2}$ with the initial condition $y(1) = 1$ using Runge-Kutta method of fourth order. Evaluate the value of y, when $x = 1.1$ and 1.2.

(c) Solve the second order differential equation $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$ with the initial condition $y(0) = 1$, $y'(0) = 0$ using Runge-Kutta method of fourth order and evaluate $y(0.1)$ and $y(0.2)$.

8. (a) State Milne's Predictor – Corrector Method to solve $\frac{dy}{dx} = f(x, y)$ subject to the initial condition $y(x_0) = y_0$.

(b) Applying Taylor series method for $\frac{dy}{dx} = x(x^2 + y^2)e^{-x}$ with the initial condition $y(0) = 1$, show that $y(0.1) = 1.0047$, $y(0.2) = 1.01813$ and $y(0.3)=1.03975$. Hence find $y(0.4)$ correct up to four decimal places by using Milne's Predictor – Corrector Method.