00135

The Open University of Sri Lanka
B.Sc. / B.Ed. Degree Programme – Level 05
Final Examination -2017/2018
Applied Mathematics
ADU5307/ADE5307 — Numerical Methods

Duration: Two Hours

Date: 18.09.2018 **Time: 01.30 p.m. – 03.30 p.m.**

Answer Four Questions Only.

1. (a) Derive Newton- Raphson formula for solving the non-linear equation $f(x)=0$.

   (b) Show that Newton- Raphson method has quadratic convergence.

   (c) Find the root of the equation $x^3-x-1=0$, lying in the interval [1, 2], taking x_0 as 1.5, correct to four decimal places, using method of Newton- Raphson.

2. (a) Prove that

   (i) $E=\Delta+1$, (ii) $\Delta=E^{1/2}\delta$,
   (iii) $\Delta^2=\delta^2+E^{1/2}\delta^3$, (iv) $\Delta^3=E^{1/2}\delta^3+\delta^4+E^{1/2}\delta^5$,
   where Δ, δ and E are the forward difference, the central difference and the shift operators respectively.

   (b) Derive Gregory- Newton forward interpolation formula.

   (c) Hence, interpolate $f(2.5)$, given that $f(x)$ passes through the points (2, 1), (3, 2), (4, 6), (5, 24) and (6, 120).

3. (a) Derive the Simpson's One –Third Rule.

   (b) If the interval $[a, b]$ is divided into $2n$ sub intervals and corresponding ordinates are denoted by $y_0,\ y_2 \ldots\ldots y_{2n}$ then show that the error in Simpson's One –Third rule is given by $|E|<\dfrac{(b-a)h^4}{180}M$, where M is the numerically greater value of $y_0^{iv},\ y_2^{iv} \ldots\ldots y_{2n-2}^{iv}$.

(c) A car completes a lap of a race track in 84 seconds. The speed of the car at each 6-second interval is determined by using a radar gun and is given from the beginning of the lap, in meters/second, by the entries in the following table. Find the length of the lap.

Time	0	6	12	18	24	30	36	42	48	54	60	66	72	78	84
Speed	41	45	49	52	49	44	40	36	33	28	26	30	35	39	41

4. (a) Applying Taylor series method of fourth order for the differential equation $\frac{dy}{dx} = x + y$ subject to the initial condition $y(1) = 0$, evaluate $y(1.1)$ and $y(1.2)$.

(b) Applying Taylor series method of third order for the system of differential equations $\frac{dx}{dt} = x + y + t$ and $\frac{dy}{dt} = 2x - t$ subject to the initial conditions $x = 0,\ y = 1$ at $t = 1$ show that $x(t) = 2t + t^2 + \frac{5}{6}t^3$ and $y(t) = 1 - t + \frac{3}{2}t^2 + \frac{2}{3}t^3$.

5. (a) (i) Derive formula for Picard's method to solve $\frac{dy}{dx} = f(x, y)$ subject to the initial condition $y(x_0) = y_0$.

(ii) Using Picard's method, find the first-three successive approximations to solve $\frac{dy}{dx} = e^x + y^2$ with the initial condition $y(0) = 0$.

(b) Applying Modified Euler method for $\frac{dy}{dx} = x + y$ subject to the initial condition $y(0) = 0$, taking $h = 0.2$, evaluate $y(0.2)$ and $y(0.4)$.

6. (a) State fourth order Runge-Kutta algorithm to solve $\frac{dy}{dx} = f(x, y)$ subject to the initial condition $y(x_0) = y_0$.

(b) Solve $\frac{dy}{dx} = \frac{1 - xy}{x^2}$ with the initial condition $y(1) = 1$ using Runge-Kutta method of fourth order. Evaluate the value of y, when $x = 1.1$ and 1.2.

(c) Solve the second order differential equation $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$ with the initial condition $y(0) = 1,\ y'(0) = 0$ using Runge-Kutta method of fourth order and evaluate $y(0.1)$ and $y(0.2)$.