C37

The Open University of Sri Lanka
B.Sc /B.Ed. Degree Programme/ Continuing Education Programme
Final Examination-2008/20009
AMU 3182/AME 5182-Mathematical Methods I
Level 05-Applied Mathematics

Duration: Two and half hours

Date: 02-01-2009 **Tme: 9.30 a.m-12.00 noon**

Answer four questions only.

01. (a) Let B be a matrix of order n. Suppose that B has a linearly independent eigen vectors $a_1, a_2, \dots, a_n$ corresponding to the (eigenvalues) $\lambda_1, \lambda_2, \dots, \lambda_n$ respectively. Show that the general solution of the system of differential equations $\dot{X}(t) = BX(t)$ is $X = \sum_{r=1}^{n} C_r a_r e^{\lambda_r t}$,where C_r are arbitrary constants.

(b) Find the general solution of each of the system of simultaneous differential equations given below.

(i)
$$\dot{x}_1 = 4x_1 + 4x_2 - \dot{x}_2$$
$$3x_1 + x_2 = -x_3 + \dot{x}_2$$
$$\dot{x}_3 - 4x_3 = 0$$

(ii)
$$\ddot{x}_1 = 4x_1 - x_2$$
$$\ddot{x}_2 = x_1 + 2x_2$$

02. (a). (i) Solve $4x^2 \frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + 3y = 0$.

Find the particular solution for which $y=1$ and $\frac{dy}{dx} = 2$,when $x=1$.

(ii) Solve $x^2 \frac{d^2y}{dx^2} + 3x\frac{dy}{dx} - 8y = \ln^3 x - \ln x$, where $x > 0$.

(b) Describe the behavior of the solution of the following system for large values of t.

$$4\ddot{x}_1 + \ddot{x}_2 + 3\dot{x}_1 + 3x_1 + 4x_2 = \cos 2t$$
$$\ddot{x}_1 + 4\ddot{x}_2 + 3\dot{x}_2 + 4x_1 + 6x_2 = \sin 2t.$$

03. (a) State the conditions for the second order semi linear partial differential equation

$$a(x,y)\frac{\partial^2 u}{\partial x^2} - b(x,y)\frac{\partial^2 u}{\partial x \partial y} + c(x,y)\frac{\partial^2 u}{\partial y^2} = f(x,y)$$

to be classified as either hyperbolic, parabolic or elliptic.

(b) Consider the partial differential equation

$$x^2\frac{\partial^2 u}{\partial x^2} - y^2\frac{\partial^2 u}{\partial y^2} = y\frac{\partial u}{\partial y} - x\frac{\partial u}{\partial x}.$$

(i) Classify the above equation as either hyperbolic, parabolic or elliptic.

(ii) Show that the characteristics of the equations are defined by the pair of ordinary differential equations, $\frac{dy}{dx} = \pm\frac{y}{x}$,and hence find the characteristics, and sketch them in the (x, y)-plane.

(iii) Hence show that above equation can be rewritten in canonical form as $\frac{\partial^2 u}{\partial \zeta \partial \eta} = 0$, where ζ and η are the characteristic variables, and thus find the general solution of the equation.

(c) Draw a diagram to show the regions of the equation

$$y\frac{\partial^2 u}{\partial x^2} + 2\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} = 0 \text{ is}$$

(i) hyperbolic (ii) parabolic and (iii) elliptic.

04. (a) Find the general solution of the system of simultaneous differential equations.

$$\frac{dx_1}{dt} = 6x_1 - 3x_2 + e^{5t}$$

$$\frac{dx_2}{dt} = 2x_1 + x_2 + 4.$$

(b) Solve the following differential equations.

(i) $u''(x) + 2u'(x) + 5u(x) = 10$,
subject to the boundary conditions $u(-\pi/2) = 0$, $u(\pi/2) = 0$.

(ii) $2x^2u''(x) + 3xu'(x) - 6u(x) = 0$,
given that $u'(1) = 1$ and $\lim_{x\to\infty} u(x)$ is bounded.

(iii) $u''(x) + 9y = 0$,
subject to $u = \pi/4$, when $x=0$ and $u = \pi/3$,when $x=\pi/6$.

05. (a) Which of the following equations are separable?

(i) $\dfrac{\partial^2 u}{\partial x^2}-\dfrac{\partial^2 u}{\partial x \partial y}+2\dfrac{\partial u}{\partial x}=0$,

(ii) $\dfrac{\partial^2 u}{\partial x^2}+y\dfrac{\partial u}{\partial x}+x\dfrac{\partial u}{\partial x}=0$,

(iii) $\dfrac{\partial u}{\partial x}-2\dfrac{\partial u}{\partial x}=u$.

For each separable equation, write down the resulting ordinary differential equation

(b) Use the method of separation of variable to find a formal solution $u(x,y)$ of the problem which consists of the heat equation,

$2\dfrac{\partial u}{\partial t}=\dfrac{\partial^2 u}{\partial x^2}$ $(0<x<\pi,\ \ t>0)$,

subject to the boundary conditions

$u(0,t)=u(\pi,t)=0$,

$u(x,0)=2\sin 3x-5\sin 5x$.

06. (a) Use the integrating factor method to find the general solution of each of the following differential equations:

(i) $\dfrac{\partial u}{\partial x}-u\tan x=\cos x$.

(ii) $(x^2+1)\dfrac{\partial u}{\partial x}+2xu=\dfrac{x^3}{(x^2+1)}$.

(b) Find the general solution of the pair of simultaneous partial differential equations:

$\dfrac{\partial u}{\partial x}=4x^3e^y+y^2$,

$\dfrac{\partial u}{\partial y}=2xy+e^y(x^4+y^4)+4y^3e^y$.

(c) Find the equations of the characteristic curves for the partial differential equation.

$-2xy\dfrac{\partial u}{\partial x}+2x\dfrac{\partial u}{\partial y}+yu=8xy$ $(x>0, y>0)$

Hence or otherwise, find the general solution of the partial differential equation.