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The Open University of Sri Lanka
B.Sc Degree Programme/ Continuing Education Programme
Final Examination- 2010/2011
Level 04- Pure Mathematics
PMU2191/PME4191 – Vector Analysis
Duration :- Two hours

Date:- 30.12.2010 **Time:- 1.00p.m.-3.00p.m.**

Answer Four Questions only.

1. (a) Consider the equation $\left(p+\frac{a}{v^2}\right)(v-b)=ct$, where the variables p, v and t denote the pressure, volume and temperature respectively, and a, b, c are constants.

Show that $\left(\frac{\partial p}{\partial v}\right)\left(\frac{\partial v}{\partial t}\right)\left(\frac{\partial t}{\partial p}\right)=-1$.

(b) Find the Second order Taylor polynomial for the function $f(x,y)=e^x \sin ay$ about the point $(0, 0)$.

(c) Considering a suitable multivariable function, estimate the value of $2.23\sqrt{5.36^3-1.96^2}$.

2. (a) (i) Define a stationary point of a single valued function $f(x,y)$, defined over the domain D. Explain briefly how you would determine its nature.

(ii) Determine the nature of the stationary points of the function $f(x,y)=3x^2y+y^3-3x^2-3y^2+2$.

(b) Find the directional derivative of the function $\phi = x^2yz+4xz^2$ at the point $(1, -2, -1)$ in the direction of the vector $2\underline{i} - \underline{j} - 2\underline{k}$.

(c) Evaluate the scalar line integral of the vector function $\underline{F}=(3x^2-4xy)\underline{i}+(3y^2-5xy)\underline{j}$ along the path C, where C is the arc of parabola $y=x^2$ from $A=(0, 0)$ to $B=(2, 4)$.

3. (a) Use surface integrals to find the closed area bounded by $y = x^2 - 4x + 5$ and $y = -x^2 + 4x - 1$. (No credit will be given for other methods)

(b) Evaluate the surface integral $\int_S y^2 \, dA$, where S is the boundary of the triangular region enclosed by $y = 1$, $x = 2$ and $y = x + 1$.

(c) Use surface integrals to calculate the area of the quarter of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ for which $x \geq 0$ and $y \geq 0$.

4. (a) Find the moment of inertia of a solid circular cylinder of constant density ρ, radius a and height h about its axis of symmetry.

(b) Find $\int_B z^3 \sqrt{x^2 + y^2 + z^2} \, dV$, where B is the solid hemisphere, with the centre at the origin, radius 1, that lies above xy-plane.

(c) Find volume of the tetrahedron bounded by the coordinate planes and the plane $3x + 2y + 6z = 6$.

5. (a) State Gauss' divergence theorem.

(b) Use Gauss' divergence theorem to evaluate the surface integral $\oint_S grad f . \underline{n} \, dA$, where f is the scalar field $f(x, y, z) = x^4 + y^4 + z^4$ and S is the spherical surface $x^2 + y^2 + z^2 = a^2$, the vector $\underline{n}$ having its usual meaning.

(c) Prove that (i) $\underline{\nabla} \cdot \left(r \, \underline{\nabla} \left(\frac{1}{r^3} \right) \right) = 0$ (ii) $\underline{\nabla} \cdot \left(r^3 \underline{r} \right) = 6r^3$, where $\underline{r}$ and r carry the usual meanings.

6. (a) State Stoke's theorem.

(b) Verify the Stoke's theorem for the vector field $\underline{F} = -y\underline{i} + x\underline{j}$, where the surface S is the disk $x^2 + y^2 \leq a^2$ in the $z = 0$ plane, oriented by the upward unit normal $\underline{n}$.

(c) If $\underline{v} = \underline{w} \times \underline{r}$, prove that $\underline{w} = \frac{1}{2} curl \, \underline{v}$, where $\underline{w}$ is a constant vector and $\underline{r}$ carries the usual meaning.