The Open University of Sri Lanka

B.Sc. /B.Ed. Degree Programme

Final Examination – 2017/2018

Applied Mathematics – Level 04

ADU4303/APU2144 – Applied Linear Algebra and Differential Equations

DURATION: TWO HOURS.

Date: 18.04.2019 **Time: 9.30am-11.30am**

ANSWER FOUR QUESTIONS ONLY.

1. (i) Find the determinant of A where

$$A = \begin{pmatrix} 1 & \cos(\beta-\alpha) & \cos(\gamma-\alpha) \\ \cos(\alpha-\beta) & 1 & \cos(\gamma-\beta) \\ \cos(\alpha-\gamma) & \cos(\beta-\alpha) & 1 \end{pmatrix}$$

(ii) Determine the rank of each of the following matrices:

(a) $\begin{pmatrix} 2 & 1 & 3 \\ 4 & 7 & 13 \\ 4 & -3 & -1 \end{pmatrix}$ (b) $\begin{pmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{pmatrix}$.

(iii) Determine the values of λ and μ such that the following system of equations has
(a) no solution
(b) a unique solution
(c) infinitely many solutions.

$$2x-5y+2z=8$$
$$2x+4y+6z=5$$
$$x+2y+\lambda z=\mu$$

Justify your answers.

2. (i) Let $A = \begin{pmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{pmatrix}$. Use the Cayley-Hamilton theorem to find A^{-1} and A^4 .

(ii) Transform the quadratic form $10x_1^2 + x_2^2 + x_3^2 - 6x_1x_2 - 2x_2x_3 + x_3x_1$ to a canonical form, using an orthogonal transformation.

3. Solve each of the following systems of differential equations:

(i) $\dot{x}_1 = x_1 + 3x_2 - 3x_3$

$\dot{x}_2 = -3x_1 + 7x_2 - 3x_3$

$\dot{x}_3 = -6x_1 + 6x_2 - 2x_3$

(ii) $\dot{x}_1 = -10x_1 + 6x_2 + 10e^{-3t}$

$\dot{x}_2 = -12x_1 + 7x_2 - 18e^{-3t}$

(iii) $\ddot{x}_1 = x_2$

$\ddot{x}_2 = x_1$

In the above, the dots represent the derivatives with respect to a parameter t.

4. (i) Find a sinusoidal particular solution for the following system of differential equations:

$\ddot{x}_1 + 3\dot{x}_2 + 2x_1 = \sin 3t$

$\ddot{x}_2 + \dot{x}_1 = \cos 3t$

(ii) Find the general solution of the differential equation

$$x^2\frac{d^2y}{dx^2} - 5x\frac{dy}{dx} + 8y = 2x^3.$$

(iii) Suppose that u is a function of two variables x and t, satisfying the partial differential equation $\frac{\partial^2 u}{\partial t^2} - c^2\frac{\partial^2 u}{\partial x^2} = 0$, where c is a non-zero constant.

By transforming to new variables ζ and ϕ, where $\zeta = x - ct$ and $\phi = x + ct$, show that the equation can be simplified to $\frac{\partial^2 u}{\partial \zeta \partial \phi} = 0$. Hence find the general solution of the partial differential equation in terms of x and t.

5. (i) Find the equations of the characteristic curves for the following partial differential equation and determine the new variable ϕ.

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + u = 2.$$

(ii) Hence obtain the general solution $u(x, y)$ of the partial differential equation given in part (i).

(iii) Find the particular solution which satisfies the condition $u(x, y) = \sin x$ on $y = -x$.

6. (i) Show that the general solution of the equation

$$y^2\frac{\partial^2 u}{\partial x^2}-2xy\frac{\partial^2 u}{\partial x\partial y}+x^2\frac{\partial^2 u}{\partial y^2}-\frac{y^2}{x}\frac{\partial u}{\partial x}-\frac{x^2}{y}\frac{\partial u}{\partial y}=0, \quad (x\neq 0,\ y\neq 0)$$

may be found by reducing it to the standard form $\dfrac{\partial^2 u}{\partial \phi^2}-\dfrac{1}{\phi}\dfrac{\partial u}{\partial \phi}=0$,

where $\zeta = x^2+y^2$ and $\phi = y$.

Use the method of characteristics to derive these expressions for ζ and ϕ.

Hence find the general solution of the partial differential equation.

(ii) Show that the equation

$$(x^2-1)\frac{\partial^2 u}{\partial x^2}+2y\frac{\partial^2 u}{\partial x\partial y}-\frac{\partial^2 u}{\partial y^2}=0$$

is (a) hyperbolic for all (x, y) outside the region R bounded by the circle $x^2+y^2=1$,

(b) parabolic on the boundary of the region R and

(c) elliptic for all (x, y) inside R.