The Open University of Sri Lanka
B.Sc Degree Programme/ Continuing Education Programme
Final Examination- 2010/2011
Level 04- Applied Mathematics
AMU 2181/AME 4181 – Mathematical Modeling I

Duration :- Two hours

Date:- 05.01.2011 **Time:- 1.30p.m.-3.30p.m.**

Answer Four Questions Only.

1. (a) A dietitian wants to design a breakfast menu for certain hospital patients. The menu is to include two items A and B. suppose that each gram of A provides 2 units of vitamin C and 2 units of iron and each gram of B provides 1 unit of vitamin C and 2 units of iron. Suppose the cost of A is 4 Rs/gram and the cost of B is 3 Rs/gram. If the breakfast menu must provide at least 8 units of vitamin C and 10 units of iron, formulate a linear programming model to determine how many grams of each item should be provided in order to meet the iron and vitamin C requirements for the least cost. Solve the model using graphical approach and hence, determine the least cost for the breakfast menu.

(b) Find the maximum value of the following linear programming problem using the graphical method:

$$Z = 2x_1 + 3x_2,$$

subject to

$$\begin{aligned} x_1 + x_2 &\geq 3, \\ x_2 &\geq 4, \\ x_2 &\leq 10, \\ x_1 &\geq 4, \\ 5x_1 + 6x_2 &\leq 60, \\ x_1 &\geq 0. \end{aligned}$$

2. Products 1 and 2 are produced by use of three machines, P, Q and R. Each unit of product 1 requires 1 hour on machine P and 2 hours on machine Q. Each unit of product 2 requires 1 hour of time on each machine. The time available on these machines is limited to 400 hours per month on machine P, 600 hours per month on machine Q and 300 hours per month on machine R. Each unit of product I can be sold to yield a profit of Rs. 50 and each unit of product 2 can be sold to yield a profit of Rs.80.

(a) Formulate a mathematical model to solve the above problem to maximize the total profit under the given conditions.

(b) Solve the formulated model in part (a) using the simplex method and find how many units of each product should be produced each month to maximize the profit?

3. **(i)** Solve the following linear programming problems using Big-M method:

(a) Minimize $Z = 3x_1 + 5x_2,$
subject to
$$x_1 + 4x_2 \geq 60,$$
$$-2x_1 - x_2 \leq -50,$$
$$x_1,\ x_2 \geq 0.$$

(b) Maximize $Z = 2x_1 + x_2,$
subject to
$$3x_1 + x_3 = 3,$$
$$4x_1 + 3x_2 \geq 6,$$
$$x_1 + 2x_2 \leq 3$$
$$x_1,\ x_2, x_3 \geq 0.$$

(ii) Draw a graphical representation of the problem in part (a) in x_1 and x_2 space and indicate the path of the simplex steps.

4. Consider the following primal linear programme:

Minimize $Z = 2x_1 + 15x_2 + 5x_3 + 6x_4,$
subject to
$$x_1 + 6x_2 + 3x_3 + x_4 \geq 2,$$
$$-2x_1 + 5x_2 - x_3 + 3x_4 \leq -3,$$
$$x_1,\ x_2,\ x_3,\ x_4 \geq 0.$$

(a) Give the dual linear programme for the above problem.

(b) Solve the dual linear programme in (a) using the simplex method and hence, write the solution to the primal problem.

5. Find all basic solutions to the following linear programming problem:

Maximize $Z = 12x_1 + 8x_2 + 14x_3 + 10x_4,$
subject to
$$5x_1 + 4x_2 + 2x_3 + x_4 = 100,$$
$$2x_1 + 3x_2 + 8x_3 + x_4 = 75,$$
$$x_1,\ x_2,\ x_3,\ x_4 \geq 0.$$

Also, find which of the above basic solutions are

(a) basic feasible.

(b) non-degenerate basic feasible.

(c) optimal basic feasible.

6. A factory can produce four products: P_1, P_2, P_3 and P_4. Each product must be processed in each of two workshops. The processing times (in hours per unit product) are given in the following table.

	P_1	P_2	P_3	P_4
Workshop 1	3	4	8	6
Workshop 2	6	2	5	8

400 hours of labour are available in each workshop. The profit margins are 4, 6, 10, and 9 rupees per unit of P_1, P_2, P_3 and P_4 produced, respectively. Assume that everything that is produced can be sold. The factory's objective is to maximize the total profit. In order to determine the factory's objective, the following linear program was solved:

$$\text{Maximize } Z = 4x_1 + 6x_2 + 10x_3 + 9x_4,$$

$$\begin{aligned} \text{subject to } \; & 3x_1 + 4x_2 + 8x_3 + 6x_4 \le 400, \\ & 6x_1 + 2x_2 + 5x_3 + 8x_4 \le 400, \\ & x_1,\ x_2,\ x_3,\ x_4 \ge 0. \end{aligned}$$

x_1, x_2, x_3 and x_4 are the units of P_1, P_2, P_3 and P_4 produced. The optimal solution is given in the following table, where s_1 and s_2 are slack variables.

	x_1	x_2	x_3	x_4	s_1	s_2	
	0.75	1	2	1.5	0.25	0	100
	4.5	0	1	5	-0.5	1	200
-z	-0.5	0	-2	0	-1.5	0	-600

(a) How many units of P_1, P_2, P_3 and P_4 should be produced in order to maximize the profit?

(b) In which range can the profit margin per unit of P_1 vary without changing the optimal basis?

(c) In which range can the profit margin per unit of P_2 vary without changing the optimal basis?

(d) A new product P_5 with unit profit margin 8 rupees and the processing times (in hours per unit product) 2 and 5 for workshop 1 and workshop 2 respectively is produced. Find the new optimal simplex table or verify that the current solution remains unchanged.