THE OPEN UNIVERSITY OF SRI LANKA
B.Sc./B.Ed. Degree Programme, Continuing Education Programme
APPLIED MATHEMATICS – LEVEL 05
APU 3147/ APE 5147 Statistical Inference
FINAL EXAMINATION - 2012/13

Duration: Two Hours.

Date: 12/12/2013	**Time: 9.30a.m – 11.30a.m**

Non programmable calculators are permitted. Statistical tables are provided.

Answer FOUR questions only.

1.

A Computer terminal has a battery pack that maintains the configuration of the terminal. These packs must be replaced occasionally. Let X denotes the life span in years of such a battery. The density function of X is given bellow.

$$f(x;\theta) = \theta e^{-\theta x}; \quad x > 0, \theta > 0$$

Let $X_1, X_2, X_3, \ldots\ldots\ldots X_n$ is a random sample from the above population.

(a) Show that the mean and the variance of the above distribution are $\frac{1}{\theta}$ and $\frac{1}{\theta^2}$ respectively.

(b) Derive moment estimators for the mean and the variance of life span of a randomly selected battery.

(c) Derive maximum likelihood estimators for the mean and the variance of life span of a randomly selected battery.

(d) A sample which is drawn from the above population is given bellow.

5.45 2.96 2.73 0.87 6.76 2.68 6.95 4.22 2.70 8.18

(i) Estimate the moment estimators for mean and variance of life span of a randomly selected battery using part (b).

(ii) Estimate the maximum likelihood estimators for mean and the variance of life span of a randomly selected battery using part (c).

2.

It is known that the distribution of a weight of a certain product is normally distributed. The weight of the product should be 100 grams. However the actual mean and the actual variance of the above distribution are unknown. A random sample of size 10 is drawn from the above distribution. The sample is given bellow.

99.53 102.65 97.55 103.41 98.51 96.95 101.88 98.00 101.30 95.84

(i) Estimate the mean weight of a randomly selected product.

(ii) Estimate the standard deviation of weight of a randomly selected product.

(iii) Estimate the mean square error of the estimator mean weight.

(iv) Company wants to Estimate the mean weight of a randomly selected tea packet bound on error 1g with 95% of confidence. Estimate the no of products to be tested to meet the requirement of the company.

(v) The production manger of the company which produces the product says that he is confident with 95% that the weight of the product is 100 grams. Construct 95% confidence interval for the mean weight of the product. Do you agree with the production manager's comment? Justify your answer.

3.

Let $X_1, X_2, X_3, \ldots\ldots\ldots X_n$ be a random sample from a uniform distribution with density given by

$$f(x;\theta) = \frac{1}{\theta} \quad ; \; 0 \leq x \leq \theta$$

(i) Find the mean and the variance of the above distribution.

(ii) Derive a moment estimator for θ. Is the moment estimator derived by you an unbiased estimator for θ? Prove your answer.

(iii) Derive maximum likelihood estimators for mean and the variance of the above distribution.

(iv) A sample drawn from the above distribution is given in the following table. Find an estimate for mean and variance for the above distribution using the maximum likelihood estimators derived in part(iii).

0.92	0.57	1.51	4.75	2.27
1.57	4.12	1.9	0.19	0.82
0.25	3.58	2.51	3.97	3.81
4.45	2.32	1.27	0.72	3.02

4.

(a) Briefly explain the following.

(i) Point Estimation.

(ii) Interval Estimation.

(b) Aseptic packing of juices is a method of packaging that entails rapid heating followed by quick cooling to room temperature in an air free container. Such packaging allows us the juices to be stored unrefrigerated. Two machines use to fill aseptic packages are to be compared with respect to their efficiency. The following data are obtained in the number of containers that can be filled per minute. It is known that variances of number of containers that can be filled per minute by both machines are equal.

	Sample size	Sample mean	Sample variance
Machine 1	30	112.5	9.5
Machine 2	30	10.2	8.9

The manager claims that the "mean no of containers that can be filled per minute by the machine 1 and the machine 2 are same". Construct 95% confidence interval for the difference of means, of number of containers that can be filled per minute, by the machine 1 and the machine 2. Comment on the manager's claim. Clearly state any assumptions you make (if any).

5.

(a) Briefly explain the following terms.

(i) Sampling distribution

(ii) Accuracy and precision of an estimator

(iii) Properties of maximum likelihood estimators

(b) The manager of a lemonade bottling plant wants to investigate performance of a production line which has only recently been installed. The manger has selected 20 one hour periods at random and has recorded the number of crates completed in each hour by this line. The table below gives the results. Construct 95% confidence interval for the mean number of crates completed per hour by the new line. Interpret your results.

77	80	86	84	86	77	77	78	86	76
79	79	83	77	82	75	78	77	75	84

6.

(a) State whether the following statements are true or false. In each case justify your answer.

(i) All unbiased estimators are consistent estimators.

(ii) All consistent estimators are unbiased estimators.

(iii) Consistent estimators are accurate and precise only for large samples.

(b) Let X be a random variable defined on a finite population of size N and $X_1, X_2, X_3, \ldots\ldots\ldots X_n$ be a simple random sample from X. Let μ be the mean and σ^2 be the variance of the population of X.

Let $S^2 = \frac{\sum_{i=1}^{n} (X_i - \bar{X})^2}{(n-1)}$ is an estimator for σ^2

Show that $E(S^2) = \frac{N}{(N-1)}\sigma^2$

(c) In a production process of automotive crankshaft bearings, the production manager is interested in the fraction of nonconforming automotive crankshaft bearings produced. A random sample of 80 automotive crankshaft bearings were tested by the production manager and found that 15 of the bearings have surface finish that is rougher than the specifications allowed. Suppose the production manager wants to estimate the fraction of nonconforming automotive crankshaft bearings in the process with a bound on the error of 0.05 at 95% level of confidence. Find the required no of automotive crankshaft bearings to be tested to meet the above requirement.