

THE OPEN UNIVERSITY OF SRI LANKA  
 B.Sc./B.Ed. Degree Programme, Continuing Education Programme  
 APPLIED MATHEMATICS – LEVEL 05  
 AMU3189/ AME 5189 - Statistical II  
 FINAL EXAMINATION - 2012/13



**Duration: Two Hours.**

**Date: 12/12/2013**

**Time: 9.30a.m – 11.30 a.m**

Non programmable calculators are permitted. Statistical tables are provided.

Answer **FOUR** questions only.

1.

A Computer terminal has a battery pack that maintains the configuration of the terminal. These packs must be replaced occasionally. Let  $X$  denotes the life span in years of such a battery. The density function of  $X$  is given bellow.

$$f(x; \theta) = \theta e^{-\theta x}; \quad x > 0, \theta > 0$$

Let  $X_1, X_2, X_3, \dots, X_n$  is a random sample from the above population.

- (a) Show that the mean and the variance of the above distribution are  $\frac{1}{\theta}$  and  $\frac{1}{\theta^2}$  respectively.
- (b) Derive moment estimators for the mean and the variance of life span of a randomly selected battery.
- (c) Derive maximum likelihood estimators for the mean and the variance of life span of a randomly selected battery.
- (d) A sample which is drawn from the above population is given bellow.

5.45    2.96    2.73    0.87    6.76    2.68    6.95    4.22    2.70    8.18

- (i) Estimate the moment estimators for mean and variance of life span of a randomly selected battery using part (b).
- (ii) Estimate the maximum likelihood estimators for mean and the variance of life span of a randomly selected battery using part (c).

2.

An investigation was conducted to compare the dust content in the flue gases of two types of solid – fuel boilers. Thirteen boilers of type A and nine boilers of type B were used under identical fuelling and extraction conditions. Over a fixed time period, of dust amount in grams deposited in similar traps inserted in each of the twenty- two flues were measured.

Type A	73.1	56.4	82.1	67.2	78.7	75.1	48.0
	53.3	55.5	61.5	60.6	55.2	63.1	
Type B	53.0	39.3	55.8	58.8	41.2	66.6	46.0
	56.4	58.9					

Assume that these independent samples come from normal populations. Sample means of dust contents of type A and type B are 63.83 grams and 52.89 grams respectively. Sample standard deviations of the dust contents of type A and type B are 10.63 grams and 9.00 grams respectively.

- (i) Test for an equality of population variances. Use 0.1 level of significance.
- (ii) Test for an equality of population means. Use 0.1 level of significance.
- (iii) Do the dust contents in the flue gases of two types of solid – fuel boilers have same distribution? Justify your answer.

3.

Let  $X_1, X_2, X_3, \dots, X_n$  be a random sample from a uniform distribution with density given by

$$f(x; \theta) = \frac{1}{\theta} \quad ; \quad 0 \leq x \leq \theta$$

- (i) Find the mean and the variance of the above distribution.
- (ii) Derive a moment estimator for  $\theta$ . Is the moment estimator derived by you an unbiased estimator for  $\theta$ ? Prove your answer.
- (iii) Derive maximum likelihood estimators for mean and the variance of the above distribution.

- (iv) A sample drawn from the above distribution is given in the following table. Find an estimate for mean and variance for the above distribution using the maximum likelihood estimators derived in part(iii).

0.92	0.57	1.51	4.75	2.27
1.57	4.12	1.9	0.19	0.82
0.25	3.58	2.51	3.97	3.81
4.45	2.32	1.27	0.72	3.02

4.

- (a) Briefly explain the following.

- (i) - Point Estimation.
- (ii) Interval Estimation.

- (b) Aseptic packing of juices is a method of packaging that entails rapid heating followed by quick cooling to room temperature in an air free container. Such packaging allows us the juices to be stored unrefrigerated. Two machines use to fill aseptic packages are to be compared with respect to their efficiency. The following data are obtained in the number of containers that can be filled per minute. It is known that variances of number of containers that can be filled per minute by both machines are equal.

	Sample size	Sample mean	Sample variance
Machine 1	30	112.5	9.5
Machine 2	30	10.2	8.9

The manager claims that the “mean no of containers that can be filled per minute by the machine 1 and the machine 2 are same”. Construct 95% confidence interval for the difference of means, of number of containers that can be filled per minute, by the machine 1 and the machine 2.

Comment on the manager’s claim. Clearly state any assumptions you make ( if any).

5.

- (a) Briefly explain the following terms.

- (i) Sampling distribution
- (ii) Accuracy and precision of an estimator
- (iii) Properties of maximum likelihood estimators

- (b) The manager of a lemonade bottling plant wants to investigate performance of a production line which has only recently been installed. The manager has selected 20 one hour periods at random and has recorded the number of crates completed in each hour by this line. The table below gives the results. Construct 95% confidence interval for the mean number of crates completed per hour by the new line. Interpret your results.

77	80	86	84	86	77	77	78	86	76
79	79	83	77	82	75	78	77	75	84

6.

- (a) State whether the following statements are true or false. In each case justify your answer.
- All unbiased estimators are consistent estimators.
  - All consistent estimators are unbiased estimators.
  - Consistent estimators are accurate and precise only for large samples.

- (b) Let  $X$  be a random variable defined on a finite population of size  $N$  and  $X_1, X_2, X_3, \dots, X_n$  be a simple random sample from  $X$ . Let  $\mu$  be the mean and  $\sigma^2$  be the variance of the population of  $X$ .

$$\text{Let } S^2 = \frac{\sum_{i=1}^n (X_i - \bar{X})^2}{(n-1)} \text{ is an estimator for } \sigma^2$$

$$\text{Show that } E(S^2) = \frac{N}{(N-1)} \sigma^2$$

- (c) In a study of the television viewing habits of children, a developmental psychologist selects a random sample of 300 first graders - 100 boys and 200 girls. Each child is asked which of the following TV programs they like best: The PUNCHI WEERAYO, SUPERMAN, or CYNDERELLA. Results are shown in the contingency table below. Conduct a statistical test to test whether the Viewing Preferences depend on the gender. Clearly state your null hypothesis and alternative hypothesis. Use 0.05 level of significance.

	Viewing Preferences		
	PUNCHI WEERAYO	SUPERMAN	CYNDERELLA
Boys	30	65	05
Girls	60	40	100