

THE OPEN UNIVERSITY OF SRI LANKA

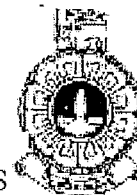
B.Sc. /B.Ed. Degree Programme, Continuing Education Programme

APPLIED MATHEMATICS-LEVEL 04

PCU2142/PSU2182/PCE4142/PSE4182- DESIGN AND ANALYSIS OF EXPERIMENTS

FINAL EXAMINATION 2012/2013

Duration: Two Hours.



Date: 03.12.2013

Time: 1.30p.m- 3.30 p.m

Answer FOUR questions only.

**Instructions:**

- This question paper consists of 06 questions and 04 pages.
- Statistical Tables are provided.
- Non-programmable calculators are permitted.
- Consider the level of significance as 0.05 for all the tests.

1) In a laboratory experiment, rabbits were used to compare the effects of 4 stimulants  $S_1, S_2, S_3$  and  $S_4$ . Thirty two rabbits from 4 different litters  $L_1, L_2, L_3$  and  $L_4$  were available for the experiment (A litter of animals is a group of animals born to the same female at the same birth. Animals of the same litter can be considered homogeneous in their characteristics than animals of different litters). There were 8 rabbits in each litter. The experimenter first randomly assigned one stimulant to each litter. Then, 3 rabbits were randomly selected from each litter and they were given the specified dose of the allocated stimulant for one week, once a day. The time consumed to complete a certain task was measured for each rabbit. Random assignment of stimulants to litters and the observed times are shown in the table below.

Stimulant	Litter	Observations
S1	L2	62 61 64
S2	L4	61 60 59
S3	L3	63 62 68
S4	L1	64 66 67

- (i) A student told that the design used in this study cannot be used to compare the stimulants. Do you agree? Justify.

- (ii) There are 20 rabbits unused for the experiment; 5 rabbits from each litter.
- What type of a design you would suggest to compare the four stimulants using the unused rabbits? Give reasons for your choice.
  - Design an experiment for this purpose. Write all the steps that you would carry out.
  - Write the model for the observations; clearly describing the parameters you use.
- (2) A motor is driven by an engine to run a machine. An experiment has been conducted to determine the effect of engine speed and engine operating temperature on the lifetime of a certain type of an electric motor that could be used in this system. For the operating temperature, there were two levels labeled as *Low* and *High*. For the operating speed, there were three levels labeled as *Low*, *Medium* and *High*. Eighteen motors of the same type were available for the study. The times to failure of motors (in months) are given below.

Temperature	Operating Speed								
	<i>Low</i>			<i>Medium</i>			<i>High</i>		
<i>Low</i>	72	74	67	67	70	74	46	54	52
<i>High</i>	82	78	86	75	84	82	64	61	57

- How many treatments are tested in this experiment? What are they?
- Write down a suitable model for the experiment. Define all the terms of the model.
- Write down the hypotheses you would test to meet the objectives of the study.

A part of the analysis of the data is given below.

Sources of Variation	Sum of Square
Temperature	480.5
Temperature*Speed	1.0
Error	186.7
Total	2312.5

- Test the hypotheses stated in part (iii), and comment on the results.
- (3) A company conducting a taxi service is interested in comparing the fuel efficiency (kilometers per gallon) achieved by four different gasoline blends (*A*, *B*, *C* and *D*). There is a possibility to make an effect on the fuel efficiency due to the differences in driving pattern of the taxi driver and the model of the taxi. The company has allocated sufficient money to use 4 drivers and 4 taxi models for this study. Assume that, you are consulted, by the company to design this study.

- (i) What is the experimental design you would suggest for this study?
- (ii) Clearly describe how you would design this study.
- (4) An experiment was carried out to investigate the effects of fertilizers on the yield of a certain variety of paddy. Three amounts of fertilizer 5lbs, 10lbs and 25lbs were tested along with the control using a completely randomized design with 5 replicates per each treatment. The yields obtained are as follows.

	Yield (bushels/plot)				
Control	5.5	4.9	4.2	2.4	5.2
5lbs	6.1	11.2	3.0	9.0	6.0
10lbs	4.2	9.7	8.1	9.5	9.2
25lbs	16.9	13.7	16.9	8.5	15.4

- (i) Identify the treatment structure of the experiment. Justify your answer.
- (ii) Write down two meaningful contrasts and test whether they are orthogonal.
- (iii) Test whether one of the contrasts that you mentioned in part (ii) is significant. Interpret the result.
- (5) An experiment was conducted to study the lifetime of four brands of tyres, say  $T_1$ ,  $T_2$ ,  $T_3$  and  $T_4$ . There are 16 tyres such that four tyres from each brand are available for the study. The experimenter suspects that the lifetime of the tyre can vary according to the type of the car used, say  $I$ ,  $II$ ,  $III$  and  $IV$ , and the position of the tyre on the car, say  $FL$ ,  $FR$ ,  $BL$  and  $BR$ . To optimize the available resources, each brand of tyre has been studied once for each type of car and once for each position of the tyre on the car. The following are the data obtained from the experiment.

Car type	Tyre position				Total
	<i>FL</i>	<i>FR</i>	<i>BL</i>	<i>BR</i>	
<i>I</i>	$T_1=31$	$T_4=76$	$T_2=40$	$T_3=54$	201
<i>II</i>	$T_2=36$	$T_3=53$	$T_4=81$	$T_1=42$	212
<i>III</i>	$T_4=76$	$T_1=43$	$T_3=62$	$T_2=38$	219
<i>IV</i>	$T_3=60$	$T_2=41$	$T_1=45$	$T_4=84$	230
<b>Total</b>	<b>203</b>	<b>213</b>	<b>228</b>	<b>218</b>	<b>862</b>

Total uncorrected sum of squares =  $\sum y^2 = 50958$

- (i) What type of experimental design is this?
- (ii) Write the corresponding model for the selected design and define all the terms of it.
- (iii) State the hypotheses to be tested in this analysis.
- (iv) A part of the analysis is given below. Complete the ANOVA table.

Source of Variation	Sum of Square
Car type	108.19
Error	29.87
Total	4715.94

- (v) Test the hypotheses and state your conclusions.
- (6) In a chemical process experiment, the effect of temperature, say  $A$ , and pressure, say  $B$ , on the reaction time has been studied. Each factor was studied at two levels and two replications were obtained. The results are given in the following table.

Treatment combination	Reaction time	
	$T_1$	$T_2$
(1)	16	19
a	17	14
b	15	13
ab	24	29

Total uncorrected sum of squares =  $\sum y^2 = 2913$ .

- (i) Estimate all the main effects and interactions.
- (ii) Analyze the data using Analysis of Variance and comment on the results.