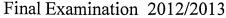
The Open University of Sri Lanka

B.Sc. / B.Ed Degree Programme / Continuing Education Programme



Applied Mathematics – Level 04

APU 2143/APE4143 - Vector Calculus

Duration:- Two Hours.

Date :- 07.06.2013

Time: - 9.30 a.m. - 11.30 a.m.

Answer Four Questions Only.

- 1. (a) Define the limit of a two variable function f(x, y) as $(x, y) \rightarrow (a, b)$; a and b being constants.
 - (b) Define the continuity of a two variable function f(x, y) at the point (a, b).
 - (c) Discuss the continuity of the following functions at the point (0, 0).

(i)
$$f(x, y) =\begin{cases} \frac{2x^2y}{x^4 + y^2} & \text{for } (x, y) \neq (0, 0) \\ 0 & \text{for } (x, y) = (0, 0) \end{cases}$$

(ii)
$$f(x, y) = \begin{cases} x \sin \frac{x+y}{4} & \text{for } (x, y) \neq (\pi, \pi) \\ \pi & \text{for } (x, y) = (\pi, \pi). \end{cases}$$

2. (a) If
$$u = y^2 e^{\frac{y}{x}} + x^2 \tan^{-1} \left(\frac{x}{y}\right)$$
, prove that

(i)
$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 2u$$
 and

(i)
$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 2u$$
 and (ii) $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = 2u$.

- (b) The base radius r of a right circular cone is increasing at the rate of 1.5mm/s while the perpendicular height h is decreasing at 6.0mm/s. Determine the rate at which the volume Vis changing when r = 12mm and h = 24mm.
- (c) A rectangular box has sides measured as 30mm, 40mm and 60mm. If these measurements are liable to be in error by $\pm 0.5mm$, $\pm 0.8mm$ and $\pm 1.0mm$ respectively, calculate the length of a diagonal of the box and the maximum possible error in the result.

- 3. (a) Prove that grad ϕ is a vector normal to the contour surface $\phi(x,y,z)=c$, where c is a constant.
 - (b) Determine the angle between the normals to the surface $xy = z^2$ at the points (4, 1, 2) and (3, 3, -3).
 - (c) If $\underline{r} = x \underline{i} + y \underline{j} + z \underline{k}$ and $r = |\underline{r}|$ then prove that

integer.

- (i) $\underline{\nabla} r = \frac{\underline{r}}{r}$, (ii) $\underline{\nabla} (\ln r) = \frac{\underline{r}}{r^2}$, (iii) $\underline{\nabla} (r^n) = nr^{n-2}\underline{r}$ where n is an
- 4. (a) State Gauss' Divergence theorem.
 - (b) Verify the above theorem for the vector field $\underline{F} = x^2 \underline{i} + z \underline{j} + y \underline{k}$ taken over the region bounded by the planes x = 0, x = 1, y = 0, y = 3, z = 0, z = 2.
 - If S is any closed surface enclosing a volume V and $\underline{A} = ax\underline{i} + by\underline{j} + cz\underline{k}$, then evaluate $\iint \underline{A} \cdot \underline{n} \, dS.$
- 5. (a) (i) State Stokes' theorem.
 - (ii) Verify Stokes' theorem for the vector field $\underline{F} = (2x y)\underline{i} + -yz^2\underline{j} y^2z\underline{k}$, where S is the surface $x^2 + y^2 + z^2 = 1, z \ge 0$.
- If \underline{a} is a constant vector and $\underline{r} = x \underline{i} + y \underline{j} + z \underline{k}$ then show that (b)
 - (i) curl $(\underline{a} \times \underline{r}) = 2\underline{a}$,
- (ii) curl $[(r, r)a] = 2(r \times a)$.
- 6. (a) Suppose S is a plane surface lying in the xy –plane bounded by a closed curve C. If $\underline{F} = P(x, y)\underline{i} + Q(x, y)\underline{j}$ then show that $\iint_{0} \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dx dy = \oint_{0} \left(P dx + Q dy \right)$ where Pand Q are functions of x and y.
 - (b) Verify the above result for the integral $\oint (3x+4y)dx xydy$, where C is the closed curve given by $x^2 + y^2 = 4$.
 - (c) Show that the vector field $\underline{F} = xy^2ze^{-x^2z}\underline{i} ye^{-x^2z}\underline{j} + \frac{x^2y^2}{2}e^{-x^2z}\underline{k}$ is irrotational. Find a scalar function ϕ such that $\underline{F} = \nabla \phi$.