The Open University of Sri Lanka
B.Sc/B.Ed. Degree, Continuing Education Programme
Final Examination 2012/2013
Level 04 Applied Mathematics
APU 2140– Statistical Distribution Theory

Duration: - Two Hours.

DATE: - 06-06-2013. Time: - 9.30 a.m.– 11.30a.m.

Non programmable calculators are permitted. Statistical tables are provided.

(1)

A team of 4 is chosen at random from 5 girls and 2 boys. Let X be the number of boys in the team

(i) Find the probability mass function of **X.**

(ii) Find the probability that the team be chosen with more girls than boys.

(iii) Using the probability mass function derived in part (i), find the expected number of girls in the team and the variance of the number of girls in the team.

(iv) Suppose the probability of giving a party by boys when both boys are selected to the team is 0.6 and probability of giving a party by girls when more girls than boys are selected to the team is 0.8. Find the probability of having a party after the section for the team.

(2)

A box containing pins has a proportion p of the pins which are out of the specifications. A random sample of n pins was drawn with replacement. Suppose X of them were out of the specifications.

(i) Derive the probability mass function of X.

(ii) Show that the moment generating function of X denoted by $M_X(t)$ is

$$M_X(t) = [pe^t + (1-p)]^n$$

(iii) Using part (ii) show that $E(X) = np$ and $var(X) = np(1-p)$.

(iv) Suppose 5% of pins are out of specifications in a box. A dealer is used to test 10 pins with replacement from a box and if he finds at least one pin with out of specifications he rejects the pins box. Find the probability that a tested pins box is accepted by the dealer.

(Hint : $\sum_{x=0}^{x=n} {}^{n}C_x \, U^x V^{n-x} = (U+V)^n$)

(3)

A company that produces a certain electrical product claims that the life time $\boldsymbol{X}$ (years) has the density function

$$f(x) = \begin{cases} kx^2 & ; \quad 0 \leq x < 1 \\ k + \frac{3}{4}(1-x) & ; \quad 1 \leq x < \frac{7}{3} \\ 0 & ; otherwise \end{cases}$$

(i) Find the value of $\boldsymbol{k}$.

(ii) Find the expected life time of a randomly selected electrical product.

(iii) Find the probability that randomly selected product will not fail within two years.

(iv) Find the cumulative distribution function of the lifetime of the product.

(v) Find the highest lifetime of the lowest 50% lifetimes of the product

(4)

(a) A restaurant kitchen has two food mixing machines $\boldsymbol{A}$ and $\boldsymbol{B}$. The average number of times $\boldsymbol{A}$ brakes down per week is 0.4 and the average number of times $\boldsymbol{B}$ brakes down per week is 0.1. Find the probability that total number of breakdowns of mixing machines $\boldsymbol{A}$ and $\boldsymbol{B}$ ***exceed*** 4, per four weeks time. You may assume that times taken to repair the machines are negligible.

(b) A factory packs belts in boxes of 200. The probability of a belt of poor quality is 0.008. What is the probability that box contains of 5 belts of poor quality.

(c) The number of bacteria on a plant follows a Poisson distribution with parameter 60. Find the probability of there being 55 to 75 bacteria on a plant.

(5)

Suppose that X_1, X_2, X_3, X_4, X_5 are independent random variables described as

$X_1 \sim N(5,4)$ $X_2 \sim N(2,9)$ $X_3 \sim exp(3)$ $X_4 \sim gamma(3,3)$ $X_5 \sim exp(3)$

Find the following probabilities. Show your calculations and state the justifications clearly

(i) $\Pr\left[\left(\frac{X_2-2}{3}\right)^2 > 3.84\right]$

(ii) $\Pr[X_3 + X_5 + X_4) < 2)$

(iii) $\Pr[2X_1 + 3X_2 + 4 > 30)$

(iv) $\Pr\left[\frac{\left(\frac{X_1-5}{2}\right)}{\left(\frac{X_2-2}{3}\right)} > 3\right]$

(6)

Entry to the ***University of ABC*** is determined by a national test. In the past years the scores on this test are normally distributed with a mean of 50 and a standard deviation of 10. In each year, only the top 30% of students based on the test get admission to the ***University of ABC.***

(i) What percent of the students scored higher than 75 in the past years?

(ii) What percentage of the students scored higher than 40 and less than 60 in the past years?

(iii) Amith wants to be admitted to this university. He took the test in this year and thinks that he could score 59. Will Amith able to get admission to ***University of ABC,*** if the scores of this year follow the same pattern as in the past?

(iii) In the past, students who had got an average above 65 at the end of the first year examination were selected to the ***Engineering programme***. According to the past records average marks of the first year examination is normally distributed with mean 47 and standard deviation 9. Find the probability that randomly selected student, who have sat for the national test is being selected for the ***Engineering programme***.

Table of probabilities of standard gamma distributions

	α									
w	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
0.2	0.47291	0.18127	0.05976	0.01752	0.00467	0.00115	0.00026	6e-005	1e-005	0
0.4	0.62891	0.32968	0.15053	0.06155	0.02297	0.00793	0.00256	0.00078	0.00022	6e-005
0.6	0.72668	0.45119	0.247	0.1219	0.05512	0.02312	0.00907	0.00336	0.00118	0.00039
0.8	0.7941	0.55067	0.34061	0.19121	0.09875	0.04742	0.02136	0.00908	0.00367	0.00141
1	0.8427	0.63212	0.42759	0.26424	0.15085	0.0803	0.04016	0.01899	0.00853	0.00366
1.2	0.87866	0.69881	0.50637	0.33737	0.20853	0.12051	0.06556	0.03377	0.01655	0.00775
1.4	0.90574	0.7534	0.5765	0.40817	0.26921	0.1665	0.09713	0.05373	0.0283	0.01425
1.6	0.92636	0.7981	0.63819	0.47507	0.33082	0.21664	0.1341	0.07881	0.04417	0.02368
1.8	0.94222	0.8347	0.69198	0.53716	0.39169	0.26938	0.17548	0.10871	0.06428	0.03641
2	0.9545	0.86466	0.73854	0.59399	0.45058	0.32332	0.22022	0.14288	0.08859	0.05265
2.2	0.96406	0.8892	0.77861	0.64543	0.50663	0.37729	0.26728	0.18065	0.11683	0.0725
2.4	0.97154	0.90928	0.81296	0.69156	0.55923	0.43029	0.31565	0.22128	0.14862	0.09587
2.6	0.97741	0.92573	0.84228	0.73262	0.60804	0.48157	0.36443	0.264	0.18346	0.12258
2.8	0.98204	0.93919	0.86722	0.76892	0.65289	0.53055	0.41285	0.30806	0.22081	0.15232
3	0.98569	0.95021	0.88839	0.80085	0.69378	0.57681	0.46025	0.35277	0.26008	0.18474
3.2	0.98859	0.95924	0.90631	0.8288	0.73078	0.6201	0.50611	0.39748	0.30069	0.21939
3.4	0.99088	0.96663	0.92145	0.85316	0.76406	0.66026	0.55	0.44164	0.34207	0.25582
3.6	0.99271	0.97268	0.93421	0.87431	0.79381	0.69725	0.59164	0.48478	0.38369	0.29356
3.8	0.99416	0.97763	0.94496	0.89262	0.8203	0.7311	0.63082	0.52652	0.4251	0.33216
4	0.99532	0.98168	0.95399	0.90842	0.84376	0.7619	0.66741	0.56653	0.46585	0.37116
4.2	0.99625	0.985	0.96157	0.92202	0.86447	0.78976	0.70135	0.6046	0.50561	0.41017
4.4	0.99699	0.98772	0.96793	0.9337	0.88269	0.81486	0.73266	0.64055	0.54406	0.44882
4.6	0.99758	0.98995	0.97325	0.94371	0.89865	0.83736	0.76139	0.67429	0.58098	0.48677
4.8	0.99805	0.99177	0.97771	0.95227	0.9126	0.85746	0.7876	0.70577	0.61617	0.52374
5	0.99843	0.99326	0.98143	0.95957	0.92476	0.87535	0.81143	0.73497	0.64951	0.55951
5.2	0.99874	0.99448	0.98455	0.9658	0.93534	0.89121	0.83298	0.76193	0.68092	0.59387
5.4	0.99898	0.99548	0.98714	0.97109	0.94451	0.90524	0.85242	0.78671	0.71033	0.62669
5.6	0.99918	0.9963	0.98931	0.97559	0.95244	0.91761	0.86987	0.80938	0.73775	0.65785
5.8	0.99934	0.99697	0.99111	0.97941	0.9593	0.92849	0.8855	0.83004	0.76319	0.68728
6	0.99947	0.99752	0.99262	0.98265	0.96521	0.93803	0.89944	0.8488	0.78669	0.71494
6.2	0.99957	0.99797	0.99387	0.98539	0.9703	0.94638	0.91185	0.86577	0.80831	0.74082
6.4	0.99965	0.99834	0.99491	0.9877	0.97467	0.95368	0.92287	0.88108	0.82813	0.76493
6.6	0.99972	0.99864	0.99578	0.98966	0.97843	0.96003	0.93262	0.89485	0.84624	0.7873
6.8	0.99977	0.99889	0.9965	0.99131	0.98164	0.96556	0.94123	0.90719	0.86272	0.80797
7	0.99982	0.99909	0.99709	0.9927	0.98439	0.97036	0.94882	0.91823	0.87767	0.82701
7.2	0.99985	0.99925	0.99759	0.99388	0.98674	0.97453	0.95549	0.92808	0.89121	0.84448
7.4	0.99988	0.99939	0.998	0.99487	0.98875	0.97813	0.96135	0.93685	0.90342	0.86047
7.6	0.9999	0.9995	0.99835	0.9957	0.99046	0.98124	0.96648	0.94463	0.91441	0.87506
7.8	0.99992	0.99959	0.99863	0.99639	0.99192	0.98393	0.97097	0.95152	0.92428	0.88833
8	0.99994	0.99966	0.99887	0.99698	0.99316	0.98625	0.97488	0.95762	0.93312	0.90037
8.2	0.99995	0.99973	0.99906	0.99747	0.99421	0.98824	0.9783	0.963	0.94102	0.91126
8.4	0.99996	0.99978	0.99922	0.99789	0.9951	0.98995	0.98127	0.96774	0.94806	0.92109
8.6	0.99997	0.99982	0.99936	0.99823	0.99586	0.99142	0.98385	0.97191	0.95433	0.92995
8.8	0.99997	0.99985	0.99947	0.99852	0.99651	0.99269	0.98609	0.97557	0.95989	0.9379
9	0.99998	0.99988	0.99956	0.99877	0.99705	0.99377	0.98803	0.97877	0.96483	0.94504
9.2	0.99998	0.9999	0.99964	0.99897	0.99752	0.99469	0.98971	0.98158	0.96919	0.95142
9.4	0.99999	0.99992	0.9997	0.99914	0.99791	0.99548	0.99116	0.98403	0.97305	0.95712
9.6	0.99999	0.99993	0.99975	0.99928	0.99824	0.99616	0.99242	0.98617	0.97645	0.96221
9.8	0.99999	0.99994	0.99979	0.9994	0.99851	0.99674	0.9935	0.98804	0.97945	0.96673
10	0.99999	0.99995	0.99983	0.9995	0.99875	0.99723	0.99443	0.98966	0.98209	0.97075
11	1	0.99998	0.99993	0.9998	0.99948	0.99879	0.99746	0.99508	0.99112	0.9849
12	1	0.99999	0.99998	0.99992	0.99978	0.99948	0.99886	0.99771	0.9957	0.9924
13	1	1	0.99999	0.99997	0.99991	0.99978	0.9995	0.99895	0.99796	0.99626
14	1	1	1	0.99999	0.99996	0.99991	0.99978	0.99953	0.99905	0.99819
15	1	1	1	1	0.99999	0.99996	0.99991	0.99979	0.99956	0.99914

Table 20.3: Let $W \sim Gamma(\alpha, 1)$ and w be a positive real number less than or equal to 15. This table contains the probabilities $\Pr(W \leq w) = \int_0^w \frac{e^{-y} y^{\alpha-1}}{\Gamma(\alpha)} dy$