The Open University of Sri Lanka
B.Sc/B.Ed. DEGREE, CONTINUING EDUCATION PROGRAMME
FINAL EXAMINATION 2014/2015
Level 05 - Applied Mathematics
APU3147/APE5147– Statistical Inference

Duration: - Two Hours.

DATE: - 09-11-2015 **Time: - 9.30 a.m. – 11.30 a.m.**

Non programmable calculators are permitted. Statistical tables are provided.
Answer four questions only.

1.

The mean weight of a certain product is an important quality characteristic that is of interest to the manufacturer. However the mean weight and standard deviation of a randomly selected product are unknown. The manufacturer is interested in finding out whether the mean is 50kg or not. A random sample of 16 products is selected, and their weights are measured. The weights of the sample products in kg are given below. Assume that weight of a randomly selected product follows a normal distribution.

48.89	52.07	49.29	51.66	52.16	49.72	48.00	49.96
49.20	48.10	47.90	46.94	51.76	50.75	49.86	51.57

(i) What is the population of interest?

(ii) Estimate the mean weight and standard deviation of a randomly selected product.

(iii) Estimate the mean squared error of the estimated mean weight found in part (ii)

(iv) Construct a 95% confidence interval for the mean weight of a randomly selected product. The manufacture clams that "mean weight of the randomly selected product is 50kg". State whether the manufacture's claim is true or false. Justify your answer.

(v) Suppose we want to estimate the mean weight of a product with a bound of 500g on the error at 95% level of confidence. Estimate no of products required to be tested to meet the above requirement.

2.

(a) Let $X_1, X_2, X_3, \ldots\ldots\ldots X_n$ be a random sample from a distribution with density given by $f(x;\theta)$. Let $\hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3, \hat{\theta}_4$, are functions of $X_1, X_2, X_3, \ldots\ldots\ldots X_n$. Suppose $\hat{\theta}_1, \hat{\theta}_2\ \hat{\theta}_3, \hat{\theta}_4$ are unbiased estimators for parameter θ and $Var(\hat{\theta}_2) < Var(\hat{\theta}_3)$. Prove each of the following statements:

(i) $Bias(\frac{\hat{\theta}_1+2\hat{\theta}_3}{4} + \frac{\hat{\theta}_2+\hat{\theta}_4}{8}) = 0$

(ii) $MSE(\ \hat{\theta}_2\ ) < MSE(\ \hat{\theta}_3\ )$.

(b) The manager of a lemonade bottling plant is interested about the performance of a production line which has only recently been installed. The manger has selected 20 one hour periods at random and has recorded the number of crates completed in each hour by this line. The table below gives the results. Assume that the number of crates completed in each hour by this line follows a normal distribution.

77	80	86	84	86	77	77	78	86	76
79	79	83	77	82	75	78	77	75	84

(i) Construct a 95% confidence interval for the variance of the number of crates completed in each hour by the new line. Interpret your results.

(ii) Suppose the mean number of crates completed in an hour by another old line is 80. The manager claims that the "mean number of crates completed in an hour by the new and the old lines are the same". Construct a 95% confidence interval for the mean number of crates completed in an hour by the new line. Comment on the manager's claim.

3.

(a) Briefly explain the following.

(i) Sampling distribution

(ii) Accuracy and precision of an estimator

(b) In a production process of automotive crankshaft bearings, the production manager is interested in the proportion θ of nonconforming automotive crankshaft bearings produced. A random sample of n automotive crankshaft bearings was drawn with replacement.

For i=1,2,.....,n, let

$$X_i = \begin{cases} 1 & ; \quad \text{If the } i^{th} \text{ item is nonconforming} \\ 0 & ; \quad \text{Otherwise} \end{cases}$$

(i) Write a suitable distribution for X_i. Clearly describe the necessary parameters in the distribution written by you if any and the assumptions if any.

(ii) Obtain the maximum likelihood estimator for θ.

(iii) Let Y = $\sum_1^n X_i$. Write a suitable distribution for Y. Hence or otherwise obtain maximum likelihood estimators for **E(Y)** and $\boldsymbol{Var(Y)}$.

(iv) Suppose total production on a particular day is 10000. Random sample of 80 automotive crankshaft bearings (drawn with replacement) were tested by the production manager. Suppose that 8 of the bearings had surface finish that nonconforming. Construct a 95% confidence interval for $\boldsymbol{\theta}$, hence construct 95% confidence interval for the total number of nonconforming automotive crankshaft bearings in the production on that day.

4.

The diameter of steel rods manufactured by two different extraction machines ***A*** and ***B*** are being investigated, Two random samples of sizes $n_A = 11$ and $n_B = 13$ were selected from the production of machines ***A*** and ***B***. The sample means and the sample variances are $\bar{x}_A = 8.73\ mm$, $s_A{}^2 = 0.35\ \ mm^2$, $\bar{x}_B = 8.68\ mm$, $s_A{}^2 = 0.40\ \ mm^2$ respectively. Assume that the diameters of steel rods manufactured by both extraction machines follow normal distributions.

(i) Construct a 95% confidence interval for the ratio of variances of diameter of steel rods manufactured by machines ***A*** and ***B***. Interpret your answer.

(ii) Dose the data provide evidence to justify the claim "variances of diameters of steel rods manufactured by machine A and B are equal"

(iii) Construct a 95% confidence interval for the difference of means of diameters of steel rods manufactured by machines A and B. Interpret your answer.

(iv) Dose the data provide evidence to justify the claim " mean diameters of steel rods manufactured by machine A and B are equal"

5.

(a) Briefly explain the following.

(i) Point Estimation.

(ii) Interval Estimation.

(b)

Let $X_1, X_2, X_3, \ldots\ldots\ldots X_n$ be a random sample from a uniform distribution with density given by

$$f(x;\theta) = \frac{1}{1+\theta} \quad ;\ 0 \le x \le 1+\theta ; \quad \theta > 0$$

(i) Prove that he mean of the above distribution is $\frac{1+\theta}{2}$.

(ii) Derive a moment estimator for θ. Is the moment estimator derived by you, an unbiased estimator for θ? Prove your answer.

(iii) Derive maximum likelihood estimators for θ and for mean of the above distribution.

(iv) A random sample drawn from the above distribution is given in the following table.

0.49	1.38	1.95	1.28	1.80
0.21	0.05	0.48	0.49	1.40
0.59	0.23	0.26	1.65	1.43
1.56	0.16	0.82	0.37	0.80

[i] Estimate θ using moment estimator derived in part (ii).

[ii] Estimate θ and mean of the above distribution using maximum likelihood estimators derived in part (iii)

6.

(a) Suppose $\hat{\theta}$ is an estimator for parameter θ. State whether the following statements are true or false. In each case justify your answer.

(i) $\hat{\theta}$ is a consistent estimator for parameter θ implies that $\hat{\theta}$ is an unbiased estimator for parameter θ

(ii) $Var(\hat{\theta}) = \frac{\theta}{n}$ implies that $\hat{\theta}$ is a consistent estimator for parameter θ.

(iii) $E(\hat{\theta}) = \frac{n\theta+\theta^2}{n}$ implies that $\hat{\theta}$ is an asymptotically unbiased estimator for parameter θ.

(b) Assignment marks and final examination marks of a particular subject for 15 students are given below. Assume that assignment marks and final examination marks follow normal distributions. The estimated correlation coefficient of assignment marks and final marks is 0.75. Using a suitable statistical method, comment on the claim that "Expected assignment mark and expected final examination mark are the same for a randomly selected student".

Student Name	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
Assignment mark	60	47	60	56	47	27	45	61	68	62	35	53	57	25	31
Final Mark	67	54	53	49	47	35	30	77	57	54	42	60	63	42	28