00201

The Open University of Sri Lanka

B.Sc./ B.Ed. Degree/ Continuing Education Programme

Level-04 Final Examination-2014/2015

PUU 2142/PUE4142-Linear Algebra

Pure Mathematics

Duration: Two Hours.

Date: 19-05-2015.	**Time: 09.30 a.m. – 11.30 a.m.**

Answer FOUR questions only.

1. (a) Define each of the following:

    (i) Minor of a matrix.

    (ii) Co-factor of a matrix.

    (iii) Adjoint of a matrix.

    (iv) Rank of a matrix.

   (b) Find the Echelon form of the following matrix and hence find the rank:

$$\begin{bmatrix} 0 & 1 & 3 & -2 \\ 0 & 4 & -1 & 3 \\ 0 & 0 & 2 & 1 \\ 0 & 5 & -3 & 4 \end{bmatrix}$$

   (c) Find the adjoint of the matrix A where

$$A = \begin{bmatrix} 3 & -1 & 4 \\ 0 & 2 & 1 \\ 1 & -1 & -2 \end{bmatrix}.$$

   Hence, find the inverse of the matrix A.

(d) Find the rank of the matrix $\begin{pmatrix} 0 & c & -b & a' \\ -c & 0 & a & b' \\ b & -a & 0 & c' \\ -a' & -b' & -c' & 0 \end{pmatrix}$ where a, b and c are positive and

$aa' + bb' + cc' = 0$.

2. (a) Determine the non-singular matrices P and Q such that PAQ is in the normal form of A

where $A = \begin{bmatrix} 3 & 2 & -1 & 5 \\ 5 & 1 & 4 & -2 \\ 1 & -4 & 11 & -19 \end{bmatrix}$

Hence, find the rank of A.

(b) Use Cayley – Hamilton theorem to find A^{-1} where $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$. Find a matrix B

such that $B = A^8 - 5A^7 + 7A^6 - 3A^5 - 5A^3 + 8A^2 - 2A + I$.

3. (a) Let the eigen values of the matrix A be $\lambda_1, \lambda_2, \ldots \lambda_n$. Then prove that eigen values of A^{-1} are $\frac{1}{\lambda_1}, \frac{1}{\lambda_2}, \ldots \frac{1}{\lambda_n}$. i.e., the reciprocals of the eigen values of A.

(b) Find the eigen values and eigen vectors of A^T, A^{-1}, A^2 and $D = A^T + \frac{2}{5}A^{-1} - 2I$ for the

matrices A where $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$.

(c) Solve the following system of simultaneous linear equations:

(i) $3x+3y+2z=1$

$x+2y=4$

$10y+3z=-2$

$2x-3y-z=5.$

4. (a) Transform the quadratic form $8x_1^2+7x_2^2+3x_3^2-12x_1x_2-8x_2x_3+4x_3x_2$ to canonical form by orthogonal transformation.

(b) Find A^5 where A is the coefficient matrix of the quadratic form given in part (a).

(c) If $X_1=\frac{1}{3}[2 \quad -1 \quad 2]^T$ and $X_2=k[3 \quad -4 \quad -5]^T$ where $k=\frac{1}{\sqrt{50}}$, construct an orthogonal matrix $A=[X_1 \quad X_2 \quad X_3]$.

5. Solve each of the following systems by LU-decomposition.

(a)

$$x+y-z=8$$
$$4x-y+3z=26$$
$$2x+y-4z=8$$

(b)

$$x+y+z+w=5$$
$$2x+y-z-2w=3$$
$$x-y+2z+w=3$$
$$x-3y+z+w=9$$

6. (a) Prove that any square matrix A can be written as the sum of a Hermitian and a skew-Hermitian matrix.

(b) Prove that $A=\begin{bmatrix} 4 & 1-3i \\ 1+3i & 7 \end{bmatrix}$ is Hermitian matrix. Find the eigen values of A.

(c) Find the Hermitian form H for

$$A = \begin{bmatrix} 0 & i & 0 \\ -i & 1 & -2i \\ 0 & -2i & 2 \end{bmatrix} \text{ with } X = \begin{bmatrix} i \\ 1 \\ -i \end{bmatrix}.$$

(d) Prove that $C = \begin{bmatrix} \frac{i}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{i}{2} \end{bmatrix}$ is a unitary matrix. Find its eigen values.