THE OPEN UNIVERSITY OF SRI LANKA
B.Sc./B.Ed. Degree Programme, Continuing Education Programme
APPLIED MATHEMATICS – LEVEL 05
APU 3147/ APE 5157 Statistical Inference
Open Book Test - 2015/2016

Duration: One Hour.

Date: 01-10-2016	**Time: 9.00 a.m. to 10.00 a.m.**

Non programmable calculators are permitted. Statistical tables are provided.

Answer all questions.

(1)

(a) The probability density function of a random variable Xthat follows a normal distribution is given by

$$f_X(x;\mu,\sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \quad ; \ -\infty \leq x \leq +\infty$$

Let $X_1, X_2, \ldots\ldots X_n$ denote arandom sample from the above population.

Derive moment estimators for the mean and the variance of the above distribution.

(b) Past experience has indicated that the time required for a beginner to become proficient with a particular function of new software product approximately follows a normal distribution.

Time took to become proficient with this particular function of new software product for 15 randomly selected beginners are given bellow in minutes.

43.64	39.48	60.78	67.63	68.2
63.7	62.83	52.94	46.92	95.61
60.75	57.1	68.67	41.35	62.66

(i) Using Part *(a)*, estimate the mean time and the standard deviation of the time (in minutes) required for a beginner to become proficient with the particular function of new software product.Estimate the mean square error of the estimate for the mean time, given by you in part (i).

(ii) Find the sample size necessary to estimate the average time required for a beginner to become proficient with the new software to within an error bound of 5 minutes, with 95% confidence.

(2)

The random variable X denotes the lifetime of a certain type of battery. The probability density function of X is given by

$$f(x,\alpha) = \frac{1}{\alpha}e^{\left(\frac{-x}{\alpha}\right)} \quad ; \ \alpha > 0, \ \ x > 0$$

and the moment generating function of X is given by

$$M_X(t) = (1-\alpha t)^{-1} \quad ; \ \ t < \frac{1}{\alpha}$$

Let $X_1, X_2, \ldots\ldots X_n$ denote lifetimes of n randomly chosen batteries from the above population.

(i) Find the mean of the above distribution.

(ii) Derive Maximum likelihood estimator for α. Is the estimator derived by you an unbiased estimator for α? Justify your answer.

(iii) A sample drawn from the above distribution is given in the following table. Using Part(ii), estimate α. .

0.67	0.17	0.31	1.70
1.07	0.32	1.64	0.19
0.48	0.97	0.04	0.79
0.66	1.48	1.76	4.64
0.25	1.40	1.44	0.99

(iv) Estimate the root mean squared error for the estimated mean calculated in part (iii).