The Open University of Sri Lanka
B.Sc/B.Ed. DEGREE, CONTINUING EDUCATION PROGRAMME
Final Examination 2015/2016
Level 04 Applied Mathematics
APU 2140/APE 5140– Statistical Distribution Theory

**Duration: - Two hours**

**Date: - 10-07-2016** **Time: - 9.30 a.m to 11.30 a.m**

**Non programmable calculators are permitted. Statistical tables are provided.**
**Answer four questions only.**

1.

A company that produces a certain electrical product claims that the life time $X$ (in months) has the density function

$$f(x) = \frac{1}{10} e^{-\left[\frac{x-2}{10}\right]} \; ; \; x \geq 2$$

(i) Find the expected life time of X.

(ii) Find the cumulative distribution function of X.

(iii) Find the probability that a randomly selected product will not fail within 5 months.

(iv) Find the probability that a randomly selected product will fail within 5 months to 10 months.

(v) Find the probability that a randomly selected product will fail before 10 months or after 15 months.

(vi) Find the highest lifetime of the lowest 50% lifetimes of the product.

2.

A team of 3 is chosen at random from 5 girls and 3 boys. Let $X$ be the number of girls in the team.

(i) Find the probability mass function of **X.**

(ii) Find the probability that the team chosen has more boys than girls.

(iii) Find the expected number of boys in the team and the variance of the number of boys in the team.

(iv) Suppose the probability of giving a party by boys when all boys are selected to the team is 0.6 and probability of giving a party by girls when more girls than boys are selected to the team is 0.8. No party will be given by girls or boys if only two boys are selected to the team. Let Z be the event of giving a party after the selection of the team

a) Find the probability of not having a party after the selection of the team.

b) Find the joint probability mass function of X and Z.

3.

(a) Proportion $p$ of the pins in a box are out of the specifications. A random sample of $n$ pins was drawn with replacement. Suppose $X$ of them were out of the specifications. The probability mass function of $X$ is given by,

$$P_X(X = x) = {}_nC_x p^x (1-p)^{n-x}; \quad x = 0,1,2,3, \ldots \ldots \ldots . n$$

Let $M_x(t)$ be the moment generating function of X.

(i) Show that $M_x(t) = [1 + p(e^t - 1)]^n$

(ii) Using part (i), show that $E(X) = np$ and $Var(X) = np(1-p)$

(b) Batch that consists of 200 coil springs from a production process is checked for conformance to customer requirements. From the past experience, the mean number of nonconforming coil springs in a batch of 200 is 20. According to the past experience what is the probability that tested batch consists of more than 30 nonconforming coil springs.

(c) A restaurant kitchen has two food mixing machines $A$ and $B$. The average number of times $A$ brakes down per week is 0.4 and the average number of times $B$ brakes down per week is 0.1. Find the probability that total number of breakdowns for a period of four weeks **exceeds** 4. You may assume that breakdowns are immediately repaired and put back to work.

4.

Particular academic program has three levels and the average marks of ***level 1*** , ***level 2*** and ***level 3*** of a student are given by the variables $X_1$ , $X_2$ and $X_3$ respectively. The ***final average*** of a student is calculated by the formula.

$$final\ average = \frac{X_1 + 2X_2 + 3X_3}{6}.$$

From the past experience it is known that $X_1 \sim N(50,100)$ $X_2 \sim N(45,100)$ and $X_3 \sim N(40,100)$. Assume that $X_1$ , $X_2$ and $X_3$ are independent:

(i) Calculate the expected ***final average*** of a student who enrolled in the above academic programme.

(ii) Calculate the variance of ***final average*** of a student who enrolled in the above academic programme.

(iii) Write the distribution of ***final average***. Clearly state the values of necessary parameters.

(iv) Find the probability of a student obtaining ***final average*** in between 40 and 60.

(v) Suppose that 750 students are enrolled to the above programme in 2016 batch. According to the rules and regulations of the above programme, student has to get a minimum of 40 for the ***final average***, to complete the programme successfully. Calculate the expected number of students who will complete the course in 2016 batch relying on to the past experience.

5.

Suppose that $X_1, X_2, X_3, X_4, X_5, X_6$ are independent random variables described as

$X_1 \sim N(5,4)$ $X_2 \sim N(2,9)$ $X_3 \sim exp(3)$ $X_4 \sim gamma(3,3)$ $X_5 \sim exp(3)$ $X_6 \sim \chi_1^2$

Find the following probabilities. Show your calculations and state the justifications clearly. You may use the gamma table at the end of the paper wherever necessary.

(i) $\Pr\left[\left(\frac{X_2-2}{3}\right)^2 < 5.024\right]$

(ii) $\Pr[X_3 + X_5 + X_4) < 1)$

(iii) $\Pr[2X_1 + 3X_2 + 4 > 30)$

(iv) $\Pr\left[\frac{\left(\frac{X_1-5}{2}\right)}{X_6} < 6.314\right]$

6.

A certain shop sells two brand of VCR, ***A*** and ***B***. Let ***X*** denote the number of VCR machines of brand ***A*** sold per day and ***Y*** denote the number of VCR machines of brand ***B*** sold per day. The following table shows the joint probabilities, according to the past data.

| ***P(x,y)*** | | ***x*** | | |
|---|---|---|---|---|
| | | 0 | 1 | 2 |
| ***y*** | 0 | 0.10 | 0.04 | 0.02 |
| | 1 | 0.30 | 0.14 | 0.06 |
| | 2 | 0.08 | 0.2 | ***k*** |

(i) Find the value of ***k***.

(ii) Find the marginal distribution function of ***X***.

(iii) What is the expected value of total number of sales of VCR on a randomly selected day?

(iv) What is the probability that on a randomly selected day, the number of VCR machines of brand ***A*** sold is more than that of brand ***B***?

(v) On a particular day salesman of the shop has sold their first VCR of brand ***A*** at 10.00 a.m. Assume that the shop opens at 9.00 a.m. and closes at 5.00 p.m. What is the probability of no sales of brand ***B*** VCR on that day?

(vi) Find the conditional probability mass function of VCR machines of brand ***B*** sold on a randomly selected day given that VCR machines of brand ***A*** sold on that day is greater than 1.

**Left tail values of Standard Gamma Table**

**W - gamma(α,1)**

**This table contain the probabilities Pr(W ≤ w)**

| | α | | | | | |
|---|---|---|---|---|---|---|
| w | 1 | 2 | 3 | 4 | 5 | 6 |
| 1 | 0.393469 | 0.264241 | 0.080301 | 0.018988 | 0.00366 | 0.000594 |
| 2 | 0.632121 | 0.593994 | 0.323324 | 0.142877 | 0.052653 | 0.016564 |
| 3 | 0.77687 | 0.800852 | 0.57681 | 0.352768 | 0.184737 | 0.083918 |
| 4 | 0.864665 | 0.908422 | 0.761897 | 0.56653 | 0.371163 | 0.21487 |
| 5 | 0.917915 | 0.959572 | 0.875348 | 0.734974 | 0.559507 | 0.384039 |
| 6 | 0.950213 | 0.982649 | 0.938031 | 0.848796 | 0.714943 | 0.55432 |