



The Open University of Sri Lanka

Faculty of Engineering Technology

Industrial Studies Program

Final Examination- 2011/2012

AEZ3238 - Mathematics for Agriculture

Date : 16-03-2012
Time : 14:00-17:00 hours
Duration : Three (03) hours

Please read the instructions below before you start answering the questions

Instructions

1. This question paper consists of two sections, **Section A** and **Section B**.
2. **SECTION A:** It contains fourteen (14) questions. Answer **all** questions. Section A carries 40 marks. Spend about one hour to answer the questions in the Section A.
3. **SECTION B:** It contains six (06) questions. Answer any four (04) questions. Each question in this section carries 15 marks.
4. Remember to write your registration No. and your Index No. correctly on the cover page of the answer book appropriately.
5. Write the question No. to which you answer in the box of the top right hand corner on the cover page of the answer book.
6. Start answering each question in **Section B** from a fresh page of the answer book and write the relevant question No. appropriately.
7. In case of a doubt, contact the Supervisor or an invigilator in the examination hall.

SECTION A

1. Each of the following expressions has a factor $(x + p)$. Find a value of p for each expression and factories the expression completely.

a) $x^3 - 6x^2 + 11x - 6$

b) $x^3 - 7x - 6$

2. Solve the following quadratic equations.

(Hint: Factorize and then solve)

a) $x^2 - 7x = 0$

b) $x^2 + 6x + 9 = 0$

c) $2x^2 - 3x - 2 = 0$

3. Solve the following equations by completing the square.

a) $x^2 - 4x + 2 = 0$

b) $2x^2 + 8x - 5 = 0$

4. Multiply and then Simplify.

a) $(3x^2 + 2x + 4)$ by $(4x^3 + 7x + 6)$

b) $(4x^6 - 2x + 1)$ by $(3x^{-6} - 2x^{-1} + x)$

5. Divide $(-4x^4 - 5x^2 + 5x + 4)$ by $(2x + 1)$

6. Solve the following two simultaneous equations.

$$(x - 2y = 7)$$

$$(x^2 + 4y^2 = 37)$$

7. Convert the following to degrees if given in radians, and to radians if given in degrees.

a) $\frac{3\pi}{2}$ radians

b) $\frac{5\pi}{6}$ radians

c) 225°

d) 210°

8. Write each of the following as a trigonometric ratio of positive acute angle.

- a) $\cos 140^\circ$ b) $\tan 185^\circ$
 c) $\sin (260^\circ)$ d) $\sin (-194^\circ)$

9. Write down the values of the following angles leaving your answers in terms of surds, where appropriate. (Examples for surds are $\frac{1}{\sqrt{2}}$, $\sqrt{3}$, etc.)

- a) $\cos 150^\circ$ b) $\tan 225^\circ$ c) $(\cos (-300^\circ))$
 d) $\sin (405^\circ)$ e) $\cot 150^\circ$

10. Prove the following identities.

- a) $\frac{1}{\cot A + \tan A} \equiv \sin A \cos A$
 b) $\frac{\sin \theta}{1 + \cos \theta} + \frac{1 + \cos \theta}{\sin \theta} \equiv \frac{2}{\sin \theta}$
 c) $\tan^2 30 + \tan^2 45 = 4\frac{1}{3}$

11. In each of the following y is given as a function of x or t .

(i.e. $y=f(x)$ or $y=f(t)$)

Differentiate y with respect to relevant variable.

- a) $4x^3$ b) $(x^2 - x^{-2})$ c) $(x^5 - 1)(x^5 + 8)$ d) $\frac{(x^2+1)}{(x^3+4)}$
 e) $t^2(t^{-1} - t^{-2})$ f) $\frac{(t^3-1)}{2t}$

12. Given that $y = x^{\frac{3}{2}} + 3x^{5/2}$, find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$

13. Find the following *indefinite* integrals.

- a) $\int x^7 dx$ b) $\int x^{-3} dx$ c) $\int x^{3/2} dx$
 d) $\int (2x^2 - \frac{1}{x^2}) dx$ e) $\int \frac{(3x-2)(2x+3)}{\sqrt{2}} dx$ f) $\int \cos 2x dx$ g) $\int \sec^2 x dx$

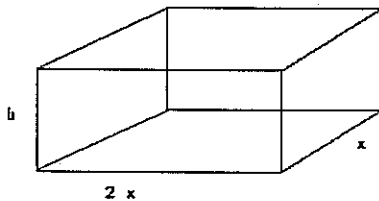
14) Evaluate the following *definite* integrals.

- a) $\int_4^9 \sqrt{x} dx$ b) $\int_{-2}^3 \{(2x - 1)(3x + 1)\} dx$ c) $\int_0^2 (4 - x^2) dx$

SECTION B

Question 1

The Sketch shown in the figure is an outline of a brick, which has the shape of a cuboid. It has a width $2x$ cm, a breadth x cm and a height h cm as shown in the figure below. The total surface area is 300 cm^2 .



- a) Show that $h = \frac{50}{x} - \frac{2x}{3}$ cm
- b) If the volume of the brick is $V \text{ cm}^3$, express V in terms of x , and any numerical coefficients associated with x .
- c) The volume varies with x , and therefore $V = f(x)$.
- (i) Find the value of x for maximum value of V .
- (ii) Hence determine the maximum value of V .
- d) The brick of the maximum volume calculated in (ii) above is crushed into powdered form, which subsequently is used to make a solid sphere. If there are no wastages, what should be the radius of the sphere?

Question 2

The curve defined by $y = 3 + 5x + x^2 - x^3$ touches the x -axis at C.

- a) Sketch the curve for the range of $-2 \leq x \leq 4$
- b) Using calculus and showing all your working clearly, find the coordinates of the turning points.
- c) Check the turning points to find out whether they are maxima or minima.
- d) Show that the point C has coordinates (3, 0)
- e) Hence calculate the area bounded by the curve and x -axis.

Question 3

- a) The radius of a soap bubble is $(4 + 5t^2)$ cm at time t seconds and it increases with time (t).
- Explain radius and surface area of the bubble as functions of t . (i.e. $r=f(t)$ and $A = \phi(t)$)
 - Find the rate of increasing radius and surface area of the bubble at $t=3$ seconds.
- b) A motor cyclist accelerates his machine at 5 m / s^2 starting from rest (i.e. $t=0$).
- What is the velocity of the motor cycle when $t = 5$ seconds.
 - What is the distance traveled by the motorcyclist from $t = 0$ to $t = 5$ seconds.
 - Obtain a relationship between velocity and distance. Hence show that when the motor cycle traveled 1000m from rest, the velocity is 100 m/s.

Question 4

A ship sails from point A to point B, a distance 5 km, on a bearing of 036° .

a) Calculate in km to two decimal places the distance by which B is:

- East of A
- North of A

b) The ship then sails to a point C, a further 8 km, on a bearing of 138°

Calculate in km to two decimal places, the distance by which C is;

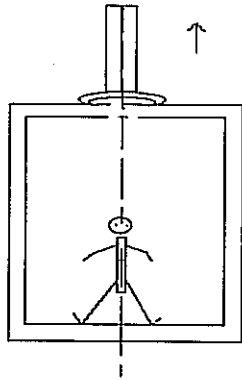
- East of A
- South of A

c) Calculate

- The bearing, to the nearest degree of A from C
- The distance, in km to two decimal places, of A from C

Question 5

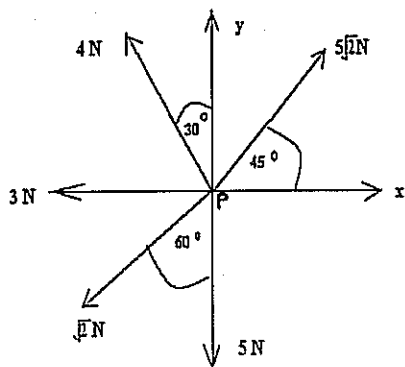
- (a) State Newton's Second Law of motion. Express it in mathematical form. Define any symbols you use in SI units.
- (b) The lift shown in the figure below with a passenger inside it is at stationery. What are the forces acting on the passenger and those acting on the lift if you consider each the passenger and the lift as a free body?



- (c) If the lift moves upwards with a uniform acceleration $f \text{ m/s}^2$, write the relevant equations to the motion of the passenger and the lift separately when Newton's Second law is applied. Define any symbols you assume.
- (d) A person of mass 70 kg is riding in a lightweight elevator (lift) of mass 500 kg. When the elevator moves upward with a constant acceleration, the cable force is 6,500 N.
- Find the value of the acceleration of the elevator.
 - If the elevator moves downwards with a constant acceleration 2.44 m/s^2 , what is the new cable force? Assume the acceleration due to gravity (g) = 10 m/s^2 .

Question 6

- What is a moment of a force?
- What is a couple?
- Find the resultant of the given system of forces below acting on point P. Find the angle the resultant makes with the x-axis.



- d) A uniform beam with 4 m long is simply supported at A and B. A being the left end of the beam, B is 3 m apart from A. The beam has a weight 20 N. An object weighting 50 N is hung at a point C, which is 1 m from the end A. Calculate the reactions at the supports A and B.
- e) The object is moved away from A to a new location along the beam. How far it can be moved from A before the beam is about to lose its stability or the reaction at A just becomes zero.

Sketch a clear digramme to solve this problem