The Open University of Sri Lanka
B.Sc. Degree Programme
Applied Mathematics - Level 04
Open Book Test-2017/2018
ADU4302/ADE4302 — Vector Calculus

DURATION: ONE HOUR

Date: 16.06. 2018   **Time: 10.30 a.m. –11.30 a.m.**

ANSWER ALL QUESTIONS.

1. (a) Find the domain and range of the function $f(x, y) = \sqrt{9 - x^2 - y^2}$.

   (b) Sketch the level curves of the above function.

   (c) Evaluate the following limits, if they exist:

   (i) $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2 - y^2}{x^2 + y^2}$,   (ii) $\lim\limits_{(x,y)\to(0,0)} \dfrac{5x^2 y^2}{x^2 + y^2}$.

   (d) If $x = r\cos\theta, \quad y = r\sin\theta$ then show that

   (i) $\dfrac{\partial r}{\partial x} = \dfrac{\partial x}{\partial r}$,   (ii) $\dfrac{1}{r}\dfrac{\partial x}{\partial \theta} = r\dfrac{\partial \theta}{\partial x}$,

   (iii) $\dfrac{\partial^2 r}{\partial x^2} = \dfrac{y^2}{r^3}$,   (iv) $\dfrac{\partial^2 r}{\partial y^2} = \dfrac{x^2}{r^3}$.

   (e) If $u = f(r)$ and $x = r\cos\theta, \quad y = r\sin\theta$ then show that
   $$\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = f''(r) + \dfrac{f'(r)}{r}.$$

2. (a) Find the equations of the tangent plane and normal line to the surface $z^2 = 4(1 + x^2 + y^2)$ at $(2,2,6)$.

   (b) Expand $e^x \ln(1 + y)$ in powers of x and y using Taylor's theorem.

   (c) Find the stationary points of the function $f(x,y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$ and determine their nature.