The Open University of Sri Lanka

B.Sc. /B.Ed. Degree Programme

Applied Mathematics – Level 05

ADU5302/ADE5302 – Mathematical Methods

No Book Test (NBT) – 2017/2018

DURATION: ONE HOUR

Date: 22.07.2018. **Time: 14:30h – 15:30h**

ANSWER **ALL** QUESTIONS.

1. (a) The Gamma function denoted by $\Gamma(p)$ corresponding to the parameter p is defined by the improper integral $\Gamma(p) = \int_0^\infty e^{-t}t^{p-1}dt, \quad (p > 0).$

(i) Prove that For a positive integer n, $\Gamma(n+1) = n!$

(ii) Compute each of the following:

$(\alpha)\ \dfrac{\Gamma(4.5)\times\Gamma 3}{\Gamma(5.5)}$

$(\beta)\ \Gamma(-4.5)$

(b) The Beta function denoted by $\beta(p,q)$ is defined by

$$\beta(p,q) = \int_0^1 x^{p-1}(1-x)^{q-1}dx,$$

where $p > 0$ and $q > 0$ are parameters.

Evaluate each of the following integrals using Beta function:

(i) $\int_0^\infty \dfrac{x^2dx}{\left(1+x^4\right)^3}$ (ii) $\int_0^{\frac{\pi}{2}} \sin^4\theta.\cos^5\theta d\theta.$

2. Let $J_p(x)$ be the Bessel function of order p given by the expansion

$$J_p(x) = x^p \sum_{m=0}^{\infty} \frac{(-1)^m x^{2m}}{2^{2m+p}.m!\Gamma(p+m+1)}$$

(a) Express $J_6(x)$ in terms of $J_0(x)$ and $J_1(x)$.

(b) Prove each of the following:

(i) $\frac{d}{dx}\{x^p J_p(x)\} = x^p J_{p-1}(x)$.

(ii) $J_0'' = \frac{1}{2}[J_2 - J_0]$.

(iii) $4J_n''(x) = J_{n-2}(x) - 2J_n(x) + J_{n+2}(x)$.