THE OPEN UNIVERSITY OF SRI LANKA

BACHELOR OF INDUSTRIAL STUDIES

TTZ5244 – QUANTITATIVE TECHNIQUES

FINAL EXAMINATION - 2009/2010

DURATION – THREE HOURS

Date: 1st April 2010	TIME: 1400-1700 hrs.

Answer Question 1 which is compulsory and any other four (04) questions. [Total questions to be answered are 05]. All questions carry 20 marks amounting to a total of 100.

You should clearly show the steps involved in solving problems.

No marks are awarded for mere answers without writing the necessary steps

Question 1

a. Solve the following equations   (02 marks)

   (i). $5^{x+2} \; 25^{x} = 125^{\,2-x}$ (ii). $2^{3x+2} + 5 = 261$

b. Express the following expressions with x as the subject.   (02 marks)

   (i) $y = A(1 + r)^{x}$ (ii) $(2 + y)\,2^{x} = 4$

   (iii) $y = e^{\,2x}$ (iv) $a^{x} = 10$ $(a>0)$

c. What do you understand by $\left[\dfrac{dy}{dx}\right]$, if y is a function of x?   (02 Marks)

d. Determine the stationary points of the following function and check whether they are maxima or minima.   (04 marks)

$$\frac{2x^{3}}{3} - 5x^{2} + 12x + 1$$

e. Determine the second derivatives of the following functions (04 marks)

(i) $y = 3x^4 + 5x^2 + 3x$ (ii) $y = \log_e x^2$

f. If $A = \begin{bmatrix} 5 & 2 \\ 3 & 7 \end{bmatrix}_{2x2}$ and $B = \begin{bmatrix} 3 & 4 \\ 2 & 1 \end{bmatrix}_{2x2}$ determine A^2 and AB (03 marks)

g. Find the determinant of the matrix A (03 marks)

$$A = \begin{pmatrix} 2 & 4 & 3 \\ 1 & 5 & 7 \\ 6 & 8 & 9 \end{pmatrix}_{3x3}$$

Question 2

a. Describe the importance of graphs in conveying the information. (04 marks)

b. Define the "gradient "and the "intercept" of a straight line graph. (02marks)

c. Publishing cost for a small book amount to Rs. 3,000/=. In addition it costs Rs. 3/= per copy to print. The publishers receive Rs. 10/= per copy from sales, but must pay 10% of this to the author, Write down an expression for the publisher's total profit in terms of the number of copies printed. Assume that all copies are sold. (04 marks)

d. Plot the graph of this expression (08 marks)

e. How many copies must be sold to break even? (Hint: Make the profit to be zero) (02marks)

uestion 3

Differentiate the following functions with respect to X

    (i) $Y = (4x - 2)(5X^2 + 8X + 4)$

    (ii) $Y = 5 \log_e (4x^2 + x + 1)$ **(04 Marks)**

b. If the price elasticity of demand E at any price and demand (P,Q) is defined as,

$$E = -\frac{dQ}{dP} \frac{P}{Q}$$ and the demand function is $Q = 200 - 2P^2$, what is the price elasticity of demand when P = 2?   **(08 Marks)**

c. Market research has revealed that just as much the sale of a soap drops with increase in price, the sales also drop when the price is too low due to customer losing confidence. The relationship between sales and price is given by the following equation

$$T = 4 + 72P - 3P^2$$

Where "P" is the price per packet of soap in rupees and "T" is the total sales.

Find at what price a packet of soap be sold so as to maximize sales. **(08 Marks)**

Question 4

A toy company incurs a fixed production cost of Rs. 1,280/= and variable cost of Rs. 80/= per toy produced. The demand function is P = 100 – Q/20, where P is the unit price and Q is the number of units (toys) of demand.

    (i) Write down an equation for the total cost of production. **(02 Marks)**

    (ii)   Express revenue as a function of Q (Revenue 'R' is given by R = PQ)

(02 Marks)

    (iii) Express the total profits as a function of Q. (Profit = revenue – total cost)

(04 Marks)

    (iv) Sketch the graph of profit as a function of Q. **(08 Marks)**

    (v) How many toys should be produced in order to maximise the profit. **(04 Marks)**

Question 5

a. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2 \times 2}$ show $A^2 - 5A = 2I$, where I is the unit matrix of order 2.

(04 Marks)

b. If $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}_{3 \times 3}$ and $B = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}_{3 \times 3}$ prove that $(AB)^T = B^T A^T$

(08 Marks)

c. Find the adjoint of A where, $A = \begin{pmatrix} 1 & 2 & -1 \\ 3 & 8 & 2 \\ 4 & 9 & -1 \end{pmatrix}_{3 \times 3}$

(08 Marks)

Question 6

ABC Apparel Company is producing three different types of garments, G_1, G_2, and G_3 at the company's two different plants P_1 and P_2. In a day P_1 produces 100, 200 and 200 garments of types G_1, G_2, and G_3 respectively. P_2 produces 120, 120, and 400 of garments of types G_1, G_2, and G_3 respectively. The monthly demand for the three types G_1, G_2, and G_3 are 5000, 6000 and 14,000 units respectively. The daily cost of operation of P_1 is Rs. 5,000/= and that of P_2 is Rs. 7,000/=

    a. Name the variable in this problem. **(02 marks)**

    b. What are the constraints of the problem? **(04 marks)**

    c. Solve the **formatted programme graphically** to determine the minimum number of days of operation per month at two plants to minimize the total cost. **(14 marks)**

Question 7

ABC Company manufactures two different products P_1 and P_2. The demand for both products is strong and as a result the firm can sell as many units of either products or of both as it can produce. Profit contribution of P_1 is Rs. 40/= and of P_2 is Rs. 50/=. The resources required to manufacture one unit of each product are shown below

	Product P_1	Product P_2
Labour hours	2	3
Machine hours	5	3

During the next month only 7,000 labour hours and 13,000 machine hours are available for production. The company's management team would like to determine the number of units of each product to be manufactured in order to maximise the total profit contribution realised on the items produced.

    (a) Formulate the problem as a Linear Programme model. **(06 Marks)**

    (b) Determine using **simplex Method,** how many of each products to be manufactured in order to maximise the profit. **(14 Marks)**

Question 8

A certain farming organization operates three farms of comparable productivity. The output of each farm is limited both by the usable acreage and by the amount of water available for irrigation. Following are the data is available for the upcoming season:

Farm	Usable Acreage	Water Available in Acre Feet
P	400	1,500
Q	600	2,000
R	300	900

The organization is considering planting three crops which differ primarily in their expected profit per acre and in their consumption of water. Furthermore, the total acreage that can be devoted to each of the crops is limited by the amount of appropriate harvesting equipment available. These amounts are as given below.

Crop	Minimum Acreage	Water Consumption in Acre feet per Acre	Expected Profit per Acre in LKR
A	700	5	400/=
B	800	4	300/=
C	300	3	100/=

Any combination of the crops may be grown at any of the farms. The organization wishes to know how much of each crop should be planted at the respective farms in order to maximize expected profit. Formulate this as a liner programming problem.

a. What are the variable in this problem?   (05 marks)

b. Write the objective of this problem.   (05 marks)

c. What are the constraints of the problem?   (10 marks)