The Open University of Sri Lanka
Faculty of Engineering Technology

Study Programme	:	**Bachelor of Industrial Studies in Agriculture**
Name of the Examination	:	**Final Examination**
Course Code and Title	:	**AEZ3238 Mathematics for Agriculture**
Academic Year	:	**2012/2013**
Date	:	16 – 08 - 2013
Time	:	09.30 – 12.30 hrs
Duration	:	3 hours

General instructions

1. Read the instruction carefully before answering questions.
2. This question paper consists of 8 questions. All questions carry equal marks.
3. Answer any 5 questions only.
4. Write question No. to which you answer on the cage provided at the top right corner of the answer book.
5. Start answering each question on a fresh page and write the Question No. at the top of the page appropriately.
6. All calculations shall be shown clearly, and any information or calculations related to an answer that does not require the attention of the examiner shall be cancelled by drawing a line, or not be submitted such information.
7. Underline the answer/answers of each and every question that you submit.
8. Do not use calculators and Logarithmic tables.

Question 1

a) Simplify the following expressions.

i) $(64)^{-2/3}$

ii) $7(x-1)^{-1} - 7x(x-1)^{-2}$

b) Expand the following expressions.

i) $(x-3y)^3$

ii) $3x(x+2)^2(2x-1)$

c) When analyzing the motion of an object, it resulted the following expression $\frac{(d+6)}{(d^2+8d+25)}$.

i) Find the reciprocal of the expression.

ii) Perform the division and find the quotient and the remainder.

d) Solve the following equation to find v in terms of other parameters.

$$pv^2 = RT(1-e)(v+b) - A$$

e) Factorise the following expressions.

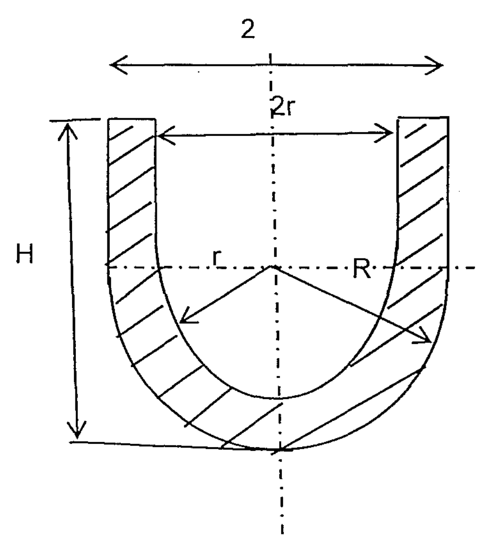
i) $x^2 - 5xy + 6y^2$

ii) $100m^2 - 200mn - 300n^2$

iii) $(a+b)^2x^2 + (a+b)^2y^2 + 2(a+b)^2xy$

Question 2

a) A hollow cylinder, of which inner radius r and outer radius R, is joined to a hollow hemisphere (inner radius r and outer radius R). Both are made of metal with a uniform thickness (R-r). When the compound object, which forms a container, is sectioned by a vertical plane through its common axis, and is viewed as shown in the figure below. Determine the volume of the metal of the container in terms of R, r and H. If the density of the material is ρ what is the mass of the container.



b) An equation relating to the number of Radium atoms (N) at any time t is given by the equation, $log_e\left(\frac{N}{N_0}\right) = -kt$, where k is a constant and N_0 is the number of atoms at t = 0. Solve for N in terms of others.

c) Solve following equations:

i) $\frac{3x}{4} + \frac{1}{6} = 2x - \frac{3}{7}$ for x

ii) $5x + 6y - 3z = 6$

$4x - 7y - 2z = -3$

$3x + y - 7z = 1$

for x, y and z

d) Combustion of Carbon usually results in the production of both carbon monoxide and carbon dioxide. For a given combustion process 1000 kg of oxygen are available. In this process it was found that nine times of oxygen is converted to carbon monoxide as oxygen converted to carbon dioxide.
   i) Obtain two simultaneous equations to the given problem.
   ii) Solve the two equations and determine how much of oxygen is converted to each of the compounds (ie. carbon monoxide and carbon dioxide).

Question 3

a) Solve $2x^2 + 12x - 16 = 0$ by completing the square.

b) A certain machine part has a rectangular section. Its length is 6 mm longer that its width. If the area of the section is 280 mm^2, what are the dimensions of the section? [Hint: Write a quadratic equation taking x as the length, which is an unknown. Then solve for x]

c) Consider the equation given below.

$(a^2 + b^2)x^2 + 2(a + b)x + 2 = 0$

i) If α and β are the roots of the given equation, determine (α+β) and (αβ).

ii) Of this equation, if $a \neq b$, are the roots real or imaginary or equal?

iii) What are the values of the roots if $a = b$?

Question 4

a) Prove the following trigonometric equations.

i) $\frac{Sin^2A - Cos^2A}{Sin^4 - Cos^4A} = 1$

ii) $Tan\left(\theta + \frac{\pi}{4}\right) = \frac{1 + Tan\,\theta}{1 - Tan\,\theta}$

b) i) Expand Sin (A+B) and Cos (A+B) in terms of Sin A, Sin B, Cos A and Cos B.

iii) Hence obtain an expression for Tan (2A) in terms of Tan A.

c) Without using logarithmic tables and calculators find the value of

i) Tan 15^0 if the value of Tan 30^0 = $(1/\sqrt{3})$.

ii) Sin 75^0 and Cos 75^0 if the value of Sin 45^0 = $(1/\sqrt{2})$ and sin 30^0 = (1/2). Leave the answer as an irrational number.

Question 5

a) Express the following angles in Radians.

210^0, 450^0, 315^0, 50^0

b) Express following angles in Degrees
$(\pi/5)$, $(2\pi/3)$, $(5\pi/4)$

c) Without using calculators and tables for trigonometric ratios, find the values of the following.
Sin 60^0, Cos (-60^0), Sin 150^0, Tan 135^0, Sin $(360^0 - 30^2)$

Question 6

a) Differentiate with respect to x, the function, F(x) = $\frac{x^2}{7} + \frac{x}{7} + \frac{1}{4}$

b) Find $\frac{dy}{dx}$, when y = $\sqrt[3]{x} + \sqrt[3]{x} + \frac{2}{x}$

c) Differentiate $(x^2 + 1)(x^2 + 2)$, and $\frac{1}{(x^2+2)}$ with respect to x

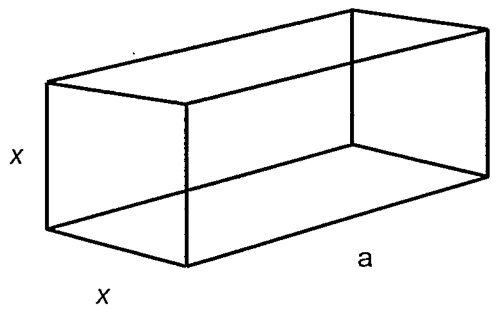
d) Find the derivative of $\frac{at^2+1}{bt^2-1}$ with respect to t

Question 7

Figure below shows an outline of a box (without the lid) with a shape of rectangular parallelepiped made out of wood. The cost of fabrication of the box is Rs.100.00 per square meter measured on the inside surface. The box is to be made, so that the fabrication cost is to be minimum and the volume of the box to be 216 m^3.
Write equations for cost of fabrication and volume of the box in terms of *a* and *x*, where *a* and *x are* inside dimensions of the box as shown in the figure.
Eliminate *a* and express the cost as a function of only *x*

Determine the values for *a* and *x* for the cost of fabrication to be a minimum.

a) Hence determine the minimum cost of fabrication.



Question 8

a) Find the Indefinite integrals of the following:

i) $\int (7x^3 - \frac{3}{x^2})\, dx$

ii) $\int (2x + 7)^{\frac{5}{2}}\, dx$

b) Evaluate the definite integral $\int_1^3 \frac{t}{(11+7t^2)}\, dt$ using the method of substitution.

c) A farm is to be set up on a given piece of land. Two coordinate axes o-x and o-y are established on the land and the perimeter of the farm is the locus that satisfies the equation, $y = 2x - x^2$. Area A_1, which is enclosed by the above locus and o-x, is selected for the farm. It was then decided to extend the farm by adding an area (A_2) to the opposite side of o-x taking A_2 as the mirror image of A_1 on the o-x. The total area of the farm is (A_1+A_2).

i) Plot the shape of the farm with respect to o-x and o-y.

ii) In order to determine the theoretical volume of soil required to fill the farm, the surface area of the farm is to be known first. Considering that the solution involves definite integral (integration within limits), calculate the total surface area (A_1+A_2) of the farm.