The Open University of Sri Lanka
B.Sc./B.Ed. Degree Continuing Education Programme
Final Examination-2008/2009
PMU 2192/ PME 4192- Linear Algebra
Pure Mathematics

Duration: Two and Half Hours.

Date: 08-01-2009.	**Time: 09.30 a.m. – 12.00 noon**

Answer FOUR questions only.

(1) (i) Define each of the following:

(a) a spanning set,

(b) a subspace,

(c) a basis.

(ii) Give an example for a spanning set which is not a basis.

(iii) Let $W = \{(a, b, c, d) \mid a + b = 0, c = 2d\}$.

Show that W is a subspace of $\mathbb{R}^4$. Find a basis for W and the dimension of W.

(2) (i) Let S be any finite subset of the vector space $\mathbb{R}^3$. Prove that the span of S is a subspace of $\mathbb{R}^3$.

(ii) Determine whether each of the following sets of vectors is linearly independent or dependent:

(a) $\{(1, 1, 2), (1, 2, 5), (5, 3, 4)\}$ in $\mathbb{R}^3$.

(b) $\{(1, 2, -1, 1), (-3, 1, 2, -1), (-3, 8, 1, 1)\}$ in $\mathbb{R}^4$.

(c) $\{(1, 1, 1), (0, 1, 1), (0, 0, 1)\}$ in $\mathbb{R}^3$.

(iii) Which of the following are linear transformations? Justify your answer.

(a) $T : \mathbb{R}^2 \to \mathbb{R}^3$

$T(x, y) = (x + 2y, x - y, y)$

(b) $T:\mathbb{R}^3\to\mathbb{R}^3$

$$T(x, y, z) = [x + y, y, (x + z)^2]$$

(c) $T:\mathbb{R}^2\to\mathbb{R}^2$

$$T(x, y) = (2x + 5y, 0)$$

(3) (i) Define *an inner product space.*

(ii) Find $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ if the set $\left\{ \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{pmatrix}, \begin{pmatrix} a \\ b \\ c \end{pmatrix} \right\}$ is an orthonormal set in $\mathbb{R}^3$.

(iii) $\{1, x, x^2, x^3\}$ is a linearly independent set of continuous functions on the interval $[-1, 1]$. Find the corresponding othonormal set starting from the given set.

(4) (i) Find the rank of the matrix $\begin{pmatrix} 0 & c & -b & a' \\ -c & 0 & a & b' \\ b & -a & 0 & c' \\ -a' & -b' & -c' & 0 \end{pmatrix}$ where a, b and c are positive and $aa' + bb' + cc' = 0$.

(ii) Let $A = \begin{pmatrix} 1 & -2 & 3 \\ 2 & -3 & 4 \\ 4 & 2 & 1 \end{pmatrix}$. Find the inverse of A.

(iii) Let $A = \begin{pmatrix} 4 & 2 & 1 & 3 \\ 6 & 3 & 4 & 7 \\ 2 & 1 & 0 & 1 \end{pmatrix}$. Find non-singular matrices P and Q such that

PAQ is of the normal form.

Hence determine the rank of A.

(5) (i) If A is a square matrix of order n and rank $(n-1)$, show that Adj A is not equal to the null matrix.

(ii) Show that if two matrices A and B have the same order and same rank, then there exist non-singular matrices R and S such that $B = RAS$.

(iii) Solve the following homogeneous linear equations in x, y, z, t given that a, b, c are all distinct.

$$(b-a)y + (c-a)z + (b+c)t = 0$$

$$(a-b)x + (c-b)z + (c+a)t = 0$$

$$(a-c)x + (b-c)y + (a+b)t = 0$$

$$(b+c)x + (c+a)y + (a+b)z = 0.$$

(6) (i) If k is a non-zero scalar, prove that the characteristic roots of kA are k times the characteristic roots of A.

(ii) Let $A = \begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix}$. Using the Cayley Hamilton Theorem, express $(5A^5 - 3A^4 + A^2 - 5I)$ in the form $\alpha A + \beta I$, where α and β are to be determined.

(iii) Let $A = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$. Find an orthogonal matrix P such that $P'AP$ is a diagonal matrix, where P' is the transpose of P.