

The Open University of Sri Lanka
 B.Sc /B.Ed. Degree Programme/ Continuing Education Programme
 Final Examination-2008/20009
 AMU 2183/AME 4183-Mathematical Modelling III
 Level 04-Applied Mathematics



064

Duration: Two and half hours

Date:12-01-2009

Time: 9.30 a.m-12.00 noon

Answer four questions only.

01. Bacteria are placed in a nutrient solution and allowed to multiply. Food is plentiful but space is limited, so competition for space will force the bacteria population to stabilize at some constant level M .

Determine an expression for the population at time t if the growth rate of the bacteria is jointly proportional to the number of bacteria present and the difference between M and the current population.

- (a) If N_0 is the initial population of the bacteria culture, show that the number of bacteria present at time t is given by

$$N(t) = \frac{MN_0}{N_0 + (M - N_0)e^{-kMt}}$$

where k is a constant.

- (b) Let N_1 and N_2 be the values of $N(t)$ when $t=T$ and $t=2T$ respectively. Show that

$$M = \frac{N_1[N_2(N_1 - N_0) - N_0(N_2 - N_1)]}{N_1^2 - N_0N_2}$$

02. A uniform light beam PQ of length a is simply suspended in a horizontal position at its both ends and a concentrated load W is kept at distance b ($b < a/2$) from the end P .

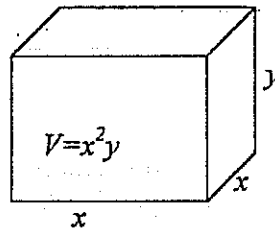
Show that the maximum deflection of the beam is given by

$$\frac{W(a-b)[b(2a-b)]^{3/2}}{9\sqrt{3}Ka}$$

and it occurs at a distance $\left\{\frac{b(2a-b)}{3}\right\}^{1/2}$ from the end P ,

where K is the flexural rigidity of the beam.

03. (a) Find the dimensions of the rectangle that, for a constant perimeter, will have the largest area.
- (b) A box having a square base and an open top is to contain 108 cubic meter. What should its dimensions be so that the cost of the material to make it will be a minimum?



04. A private phone vendor has introduced a special offer to sell his mobile phones. He has a total of 47 Nokia and Ericsson phones. Any person purchasing a Nokia phone will receive 2 more KIT cards than he had purchased an Ericsson phone. However an Ericsson phone cost 900/= rupees less than a Nokia phone. The total cost of phones is 390900/= rupees and the total number of KIT cards on offer is 195. Hence, find
- The number of Nokia and Ericsson phones.
 - The number of KIT cards offered to a person who purchases a Nokia phone.
 - The number of KIT cards offered to a person who purchases an Ericsson phone.
 - The cost of a Nokia phone and the cost of an Ericsson phone.

05. Newton's law of cooling states that the rate at which a body cools is proportional to the difference between the temperature of the body and that of the medium in which it is situated.
- A body of temperature $80^{\circ}F$ is placed at time $t=0$ in a medium the temperature of which is maintained at $50^{\circ}F$. At the end of 5 min, the body has cooled to a temperature of $70^{\circ}F$.
- What is the temperature of the body at the end of 10 min.?
 - When will the temperature of the body be $60^{\circ}F$?

06. A company is planning to buy certain items required for a production from an outside supplier at a cost of Rs.100.00 per unit. The manufacturing department estimates an annual usage of 35, 280 units. The purchasing cost per order is Rs.170. It was found that yearly carrying cost is Rs.17 per unit of inventory. Assume that shortages are not allowed. Write the total Inventory Cost as a function of order quantity Q .
Hence, find

- (i) The Economic Order Quantity.
- (ii) The minimum Total Inventory Cost.
- (iii) The number of orders per year.

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