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THE OPEN UNIVERSITY OF SRI LANKA
B.Sc/B.Ed Degree Programme/Continuing Education Programme
APPLIED MATHEMATICS - LEVEL 04
AMU 2182/ AME 4182 – Statistics I
FINAL EXAMINATION 2008/2009

DURATION: TWO AND HALF HOURS.

DATE: 06 – 01 – 2009	TIME: 9.30am – 12.00noon

ANSWER **FOUR** QUESTIONS ONLY.

Statistical Tables are provided. Non programmable calculators are permitted.

1. A pack of cards used for a game consist of 12 cards. Each card has a coloured dot in the middle and a number printed. The colours are blue, green and red and each colour appears in exactly four cards. The numbers appearing on the four blue cards are 1, 2, 3 and 4. The number appearing on each of the green card is 6. The number appearing on each of the red card is 9. A single trial of the game is defined as randomly drawing a card from the pack and recording the colour and the number printed.

   i) Write down the sample space Ω.

   ii) What is the probability that the card has a blue dot?

   iii) What is the probability that the number printed on the card is even?

   iv) Given that the card drawn has a blue dot, what is the probability that the number printed on the card is even?

   v) Given that the card drawn does not have a blue dot, what is the probability that the number printed on the card is even?

   vi) What is the probability that the card drawn has a blue dot with an even number or a red dot with an even number?

2. Suppose that the actual weight of a randomly chosen packet of sugar labeled $1kg$ is normally distributed with a mean of 990 grams and a standard deviation of 15 grams.

   i) What is the probability that the actual weight of a randomly chosen packet will be less than the labeled weight?

   ii) If 50 packets are selected randomly, what is the expected number of packets that will have a weight not less than the labeled weight?

   iii) Suppose a quality controller carefully examined the weights of 1000 randomly selected packets and finds that 700 of the packets have weights less than the required minimum weight of m grams. Estimate the value of m.

   iv) Suppose the quality controller instructs to add 8 grams of sugar to each packet. What proportion of packets will then have a weight less than the labeled weight of 1 kg?

3. :Let X be a random variable that follows a geometric distribution with success probability p and probability density (mass) function $f_X(x)$ given by

$$f_X(x)=\begin{cases} p(1-p)^x, x=0,1,2,\cdots \\ 0, \text{ otherwise.} \end{cases}$$

i) Verify that $f_X(x)$ is a probability density (mass) function.

ii) Find the moment generating function $M_X(t)$ of X.

iii) Using part (ii) or otherwise, find the mean of X.

iv) The six sides of a die are marked with 2, 2, 2, 3, 4, and 5. Suppose the sides marked with odd numbers have the same chance of facing up. The sides marked with even numbers also have the same chance of facing up and is twice as that for sides with odd numbers. An odd number facing up is regarded as a win. Using part (iii) or otherwise, find the expected number of times one has to roll the die to win, if he decides to roll the die until he wins. Consider the outcome on each trial as independent of what happened in the past.

4. Let A and B be any two independent events. In the following expressions, let the superscript c denote the complement of the event.

a) In the usual notation, prove or disprove the following:

i) The events A and B^C are independent;

ii) $P(A|B)=P(A|B^c)$;

iii) $P(A\cap B)=P(A\cap B^C)$.

iv) $P(A\cup B)=1-P(A^c)P(B^c)$.

b) A card pack has 7 cards marked with 1, 2, 3, 4, 4, 5 and 5. A card is picked randomly.

Let A be the event that the card picked has an even number.

i) Give an example of an event B such that A and B are mutually exclusive.

ii) Give an example of an event B such that A and B are exhaustive but not mutually exclusive.

5. A machine has two batteries with lifetimes (in years) X and Y. The machine can work properly with a single battery but when both batteries are dead, the machine fails.

Suppose X and Y have the joint density function $f(x,y)$ given by

$$f(x,y) = k(x+y^2), 0 \le x \le 1; 0 \le y \le 1.$$

i) Find the value of k.

ii) Find the marginal density functions of X and Y.

iii) Are X and Y independent? Give reasons for your answer.

iv) Assume that the machine starts work with one of the batteries and once it fails it automatically uses the other battery. The machine may fail due to failure in batteries or due to other reasons. Suppose at the start, both batteries are new and until used batteries remain as new. The expected lifetime of the machine when at least one battery is working is 4 years. Estimate how long the machine can work when two new batteries are installed.

6. An intensive care unit has 5 beds. Let X denote the total number of beds vacant at a randomly chosen time point.

The management has made the following observations:

- Around 2% of the time no bed is vacant.
- Around 3% of the time all beds are vacant.
- The chances that only one bed is vacant or only two beds are vacant are nearly the same.
- Around 40% of the time at most two beds are vacant.
- Around 30% of the time at least 4 beds are vacant.

i) Find the probability that exactly three beds are vacant.

ii) Write down the probability mass function of X.

iii) Find the expected number of beds vacant at a given instant.

iv) Find the variance of X.

v) If four patients are waiting to be admitted to the intensive care unit, find out the expected number of patients who will fail to find vacant beds?