00268

**THE OPEN UNIVERSITY OF SRI LANKA**
**DIPLOMA IN TECHNOLOGY / BACHELOR OF SOFTWARE ENGINEERING – LEVEL 04**
**FINAL EXAMINATION – 2013/2014**
**MPZ4140 / MPZ4160 - DISCRETE MATHEMATICS I**
**DURATION – THREE HOURS**

**DATE: 24th August 2014** **TIME: 1330hrs – 1630 hrs.**

**Instructions:**

- **Answer only six questions.**
- **Please answer a total of six questions choosing at least one from each section A, B, and C.**
- **State any assumption that you made.**
- **All symbols are in standard notation.**

## SECTION – A

01\. i. What is meant by a "statement"? Which of the following are statements and find their truth values?

a) "9 + 3 =13".
b) "Where are you going ?"
c) "Kamala is a beautiful girls in Bio-Maths class".
d) "Please give me a cup of tea."

[25%]

ii. Determine the truth value of the following propositions:

a) "$\sqrt{2}$ is a rational number or $\sqrt{2}$ is a prime number".
b) "If 3+ 3 =9, then 3 + 3 =6 and 3 +3 = 5 ".

[20%]

iii. Let $p, q, r$ be three statements.
Verify that $[(p \rightarrow q) \wedge (r \rightarrow q)] \leftrightarrow [(p \vee r) \rightarrow q]$ is a tautology or not.

[20%]

iv. Prove De Morgan's Laws by using truth tables.

[20%]

v. Use the laws of the algebra of proposition to show that,

$$(p \wedge q) \vee [\sim(\sim p \vee q)] \equiv p.$$

[15%]



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02. i. Prove that $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}$ such that $x^2 + y = 7$. [20%]

ii. State the "converse", "negation", and "contrapositive" of the following statement:
"If two sides of a triangle are equal, then the opposite angles are equal". [15%]

iii. Prove that, if $|x| > |y|$, then $x^2 > y^2$.
[15%]

iv. By giving a counter example, disprove each of the following statements:

a) $\forall x \in \mathbb{R}, \exists y \in \mathbb{R},\ x - y^2 = 15$.

b) $\forall x \in \mathbb{R},\ \forall y \in \mathbb{R},\ \frac{x+y}{2} \geq \sqrt{xy}$. [20%]

v. Test the validity of the following argument:

"If robbery was the motive for the crime then the victim had money in his pockets. But robbery or vengeance was the motive for the crime. Therefore, vengeance must have been the motive for the crime. "
[30%]

03. i. Using mathematical induction, for a positive integer *n*, prove each of the following:

a) $3^{2n} - 1$ is divisible by 8.

b) $1^3 + 2^3 + 3^3 + \cdots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$.
[50%]

ii. If $x^2$ is an odd integer, then $x$ is an odd integer. [30%]

iii. Over the universe of animals, the propositional functions *A(x), B(x) and C(x)* are defined by:
*A(x)* : *x* is a whale
*B(x)* : *x* is a fish
*C(x)* : *x* lives in water
Translate each of the following in your own words:

a) $\exists x\ (\sim C(x))$

b) $(\exists x)\ (B(x) \wedge \sim A(x))$.
[20%]

## SECTION – B

04. i. Find the elements in each of the following sets:

$A = \{x: x^2 = 9,\ x - 3 = 5\}$
$B = \{x: x^2 + 1 = 0,\ x$ is a complex number $\}$
$C = \{x: x^2 + 13 = 9,\ x$ is a real number $\}$
$D = \{x: |x + 2| \leq 4,\ x \in \mathbb{R}\}$ [20%]

ii. Without using Venn diagram, show that

$A \cap (B \oplus C) = (A \cap B) \oplus (A \cap C)$, where $\oplus$ is symmetric difference. [35%]

iii. Let $A = \{a, b, \{a, b\}, ab\}$. Find the power set of $A$. [15%]

v. Let $L = \{1,2,3,4,5\}$, $M = \{x: x \in \mathbb{N}, |x - 4| < 7\}$, and $N = \{x: x^2 - 16 = 0, x \in \mathbb{N}\}$.
With usual notation, find the element of following sets:

$L \oplus M,\ L \cap (M \oplus N),$ and $M \backslash (L \oplus N)$. [30%]

05. i. Let $A = \{1,2,3,4\}$. Find a relation of $R$ on $A$ which is

a) neither symmetric nor anti-symmetric
b) anti-symmetric and reflexive but not transitive
c) transitive and reflexive but not anti-symmetric.

[45%]

ii. Let $\mathbb{N}$ be the set of natural numbers. The relation $R$ on the set $\mathbb{N} \times \mathbb{N}$ of ordered pairs of natural numbers is defined as:

$$(a, b)R(c, d) \Rightarrow ad = bc.$$

Prove that $R$ is an equivalence relation. [30%]

iii. Show that the operation "division" on the set of natural numbers is a partial order. [25%]



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06. i. Define the terms "one to one" function and "onto" function. [10%]

a) Let $f: \mathbb{N} \rightarrow \mathbb{N}$ where

$$f(x) = \begin{cases} 2x, & \text{if } x \text{ is even} \\ x, & \text{if } x \text{ is odd} \end{cases}$$

Is $f$ one to one? Justify your answer. [25%]

b) Prove that the function $g: \mathbb{R}\backslash\{5\} \rightarrow \mathbb{R}\backslash\{0\}$ is one to one and onto, where $g(x) = \frac{2}{x-5}, x \in \mathbb{R} - \{5\}$. Hence find a formula for $g^{-1}$.

[40%]

ii. Let $f, g$ and $h$ be functions from $\mathbb{Z}$ to $\mathbb{Z}$, which are defined by
$f(x) = x + 7$,
$g(x) = x - 3$, and
$h(x) = x^3$ respectively.

Define functions $fog$, $hogof$ and $gohoh$. [15%]

iii. Let $A=\{a, b, c\}$ and $B=\{1, 2, 3\}$. Find $A \times B$ and $A^2$.

[10%]

## SECTION – C

07. i. Let $a, b, c$ be any integer numbers. Show that,

a) If $a|b$ and $a > 0, b > 0$, then $|a| \leq |b|$.

b) If $a|b$, and $a|c$, then $a|(b + c)$ and $a|(3b - 2c)$.

c) If $a|(3x^2 - 2x + 3)$, then $a|[x^2(9x^2 - 12x + 22) - 3(4x + 3)]$.

[50%]

ii. Prove the following:

a) If $gcd(a, b) = 1$ and $c|a$, then $gcd(b, c) = 1$.

b) If $gcd(a, b) = 1$, $gcd(a, c) = 1$, then $gcd(a, bc) = 1$. [30%]

iii. Define a prime number.
If $n \geq 5$ is a prime number, show that $n^2 + 2$ is not prime number. [20%]



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08. i. Let $gcd(a, b) = 1$. Show that $gcd(2a + b, a + 2b) = 1$ or 3.

[20%]

ii. Let $m$, $n$ be integers. Show that
$(m, n) = \left(\frac{m-n}{2}, n\right)$ if both $m$, $n$ are odd.

[20%]

iii. Use the Euclidean Algorithm to find the integer $x$ and $y$ such that

$$37x + 249y = 1.$$

Determine all integer solutions of the following Diophantine equation:

$$37x + 249y = 5000.$$

[60%]

09. i. Let $a \equiv b(mod\ m)$. Show that $mc + a \equiv b(mod\ m)$ where $c$ is an integer.

[10%]

ii. Solve each of the following system of congruence:

a) $x \equiv 5(mod\ 6)$
$x \equiv 4(mod\ 11)$
$x \equiv 3(mod\ 17)$

b) $2x \equiv 1(mod\ 5)$
$3x \equiv 9(mod\ 6)$
$4x \equiv 1(mod\ 7)$
$5x \equiv 9(mod\ 11)$

[75%]

iii. Prove that $a \equiv b(mod\ m)$, $b \equiv a(mod\ m)$, and $a - b \equiv 0(mod\ m)$ are equivalent statements.

[15%]

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