The Open University of Sri Lanka

Faculty of Engineering Technology

Industrial Studies (Agriculture) Programme

Final Examination – 2013/ 2014

AEZ3238 Mathematics for agriculture

Date : 29-08-2014
Time : 9.30 a.m. – 12.30 p.m.
Duration : Three (03) hours

Read the instruction given below before starting to answer the question paper

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1. This question paper has **eight (08)** questions. Answer any **six (06)** questions.
2. All questions carry equal marks.
3. Answers to questions shall be legible, clear, and neat and step by step procedure is important in all calculations. Final answers should be underlined.
4. Start answering each question from a fresh page with the relevant question number written at the top of the page.
5. It is important that the candidate writes the question numbers of the attempted questions on the cover page.
6. Do not use red ink or pencils to write your answers.
7. Write your Index Number clearly on the answer book.
8. Delete by drawing a line on the page/s in the answer book, where you have material which do not require the attention of the examiner.

Question 1

a) Covert the angles given in radians below to degrees. The value of the angle should be rounded off to the nearest first decimal place and positive value.

i) 2 rad ii) $\frac{9\pi}{2}$ iii) $-\frac{\pi}{2}$

b) Convert the angles given in degrees below to radians.

i) 150^0 ii) 540^0 iii) -210^0

c) Without using calculators find,

i) $\csc\Theta$ given that $\sin\Theta = -\frac{3}{4}$

ii) $\tan\Theta$ given that $\sec\Theta = \frac{3\sqrt{2}}{4}$

iii) $\cot\Theta$ given that $\cos\Theta = -\frac{1}{4}$

Question 2

a) Prove the following trigonometric relationships. (csc = Cosec)

I) $\cot\Theta \sin\Theta = \cos\Theta$

II) $(1 + \cos\Theta)(1 - \cos\Theta) = \sin^2\Theta$

III) $(\csc\Theta + \cot\Theta)(\csc\Theta - \cot\Theta) = 1$

IV) $\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = \csc\theta \sec\theta$

V) $\frac{\tan\theta + \cot\theta}{\tan\theta} = \csc^2\theta$

b) A ship leaves port **A** at 12.00 noon and sails due west (**W**) at 20 nautical miles per hour. At 2.00 pm the ship is at **B** and changes its course (path) to **N 54^0 W**, as shown in the **Figure 1** and sails at the same speed towards port **C**. Find the ship's bearing and distance from the port **A** when the ship is at port **C** at 3.00 pm.

[*In usual notations, any triangle with sides **a**, **b** and **c** with opposite angles BAC, CBA and BCA satisfies the following trigonometric relationships.*]

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$; *and* $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$; $\cos B = \frac{a^2 + c^2 - b^2}{2ac}$;

$\cos C = \frac{a^2 + b^2 - c}{2ab}$

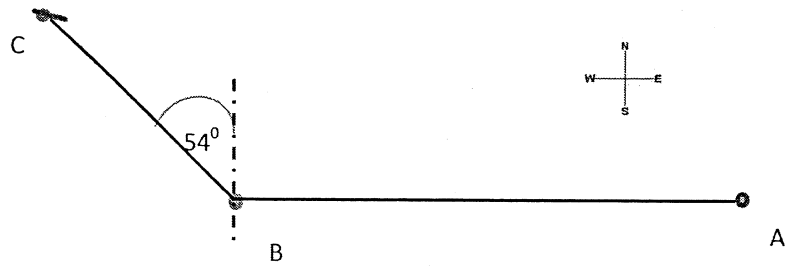

Figure 1

Question 3

a) Factorise the following expressions.

i) $3xy^2 + 12\,x^2y$

ii) $4pq^2 - 20pq + 8p^2q$

iii) $15x - 25x^2$

iv) $7x^2 + 14xy - 21y^2$

v) $27x^3 - 64y^3$

b) Solve the following equations.

i) $3x^2 = 5x - 1$ Use completing the square method.

ii) $4x^2 - 11x + 6 = 0$ Use quadratic formula.

iii) $3x^2 - 27 = 0$ Factorise the left hand side first and solve.

iv) $x + 2y = 1;\quad 2y^2 + 5x + 3 = 0$ Use substitution method.

v) $x + y - z = 4;\quad x - 2y + 3z = -6;\quad 2x + 3y + z = 7$

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Question 4

a) The equation $4x^2 + 4(k-1)x + 4 = 0$ should have real roots. Find the range of the value of k.

b) In the above equation if the value of $k = -4$, and the roots are given by α and β, find the values of the following:

i) $\alpha\beta$; ii) $(\alpha + \beta)$; iii) $(\alpha^2 + \beta^2)$; iv) $(\alpha^2 - \beta^2)$; v) $(\frac{1}{\alpha^2} + \frac{1}{\beta^2})$

c) During a cricket practice session, the motion of the cricket ball is modeled, and the height of the ball from the ground level in meters is given as $h = -5t^2 + 11t + 1$, where t is the time in seconds. How long does the ball take to reach a height of 3 m from the ground?

Question 5

a) Evaluate the limits of the following functions:

i) $\lim_{x\to 4} \frac{x^2-7x+12}{x-4}$; ii) $\lim_{x\to 1}(x^2+1)$; iii) $\lim_{x\to 0} \frac{(x+3)^2-9}{x}$

b) Differentiate following functions with respect to the relevant variable.

i) $\frac{3}{x^2}$

ii) $10\,x^{15}$

iii) $3z^2 + \frac{3}{z^2} + 10z^{15} + z + 5$

iv) $(t^2 + 2t + 1)(t^3 + 3t)$

v) $(x^2 + 1)^{10}$

vi) $\frac{t^2+2t+1)}{t^3+3t}$

c) Given that $y = x^4 + 3x^3 + 5x$, determine first derivative and second derivative.

Question 6

a) The function $f(x) = (x - 2)^2 (x + 4)^2$ has two minimum turning points and one maximum turning point.

i) What are the coordinates of these turning points?

ii) Check for maximum and minima and identify the nature of the turning points.

b) A rectangle has a constant perimeter, *P*. The length and breadth of the rectangle can be varied, and at one instant the area is maximum. Find the dimensions of the rectangle in terms of *P*, which will result the maximum area.

Question 7

a) Find the indefinite integral of the following functions.

i) $\int (x^4 + 3x - 9)\, dx$

ii) $\int (3\sqrt[4]{x^3} + \frac{7}{x^5} + \frac{1}{6\sqrt{x}})\, dx$

iii) $2\int x(x^2 + 5)^7\, dx$

b) Find the finite integral of the following functions.

i) $\int_0^2 (x^2 + 1)dx$

ii) $\int_1^2 (y^2 + y^{-2})dy$

iii) $\int_1^2 (\frac{2w^5 - w^2}{w^2})dw$

Question 8

The Figure (a) below shows the cross section of a reservoir which stores water for irrigation purposes. Figure (b) is a three dimensional sketch of the whole tank. The cross section, which is symmetrical about O-Y axis, is uniform over the tank length. The tank is 25 m long and 16 m wide. The profile of the tank wall with respect to x-y coordinate system with origin at O is given by the equation, $x^2 = 32y$. The maximum height of the tank is 2 m as shown in the figure. It is required to estimate the volume of the water that the tank can hold and therefore answer the following questions.

i. Calculate the area of the cross section of the tank in square meters. *[Hint: Use definite integration and integrate with respect to variable y.]*

ii. Hence find the water holding capacity of the tank in cubic meters.

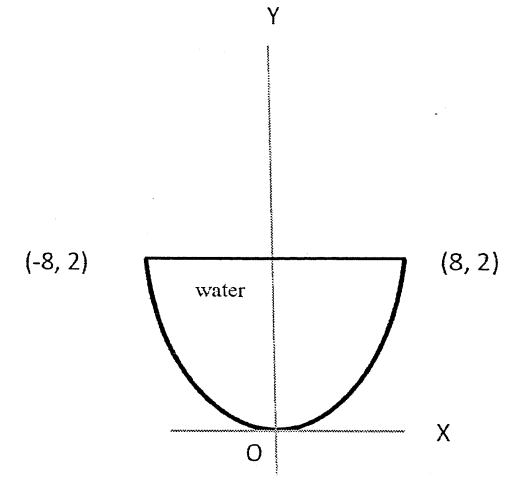

Figure (a): Sectional profile of the Tank with respect to O-X and O-Y coordinate

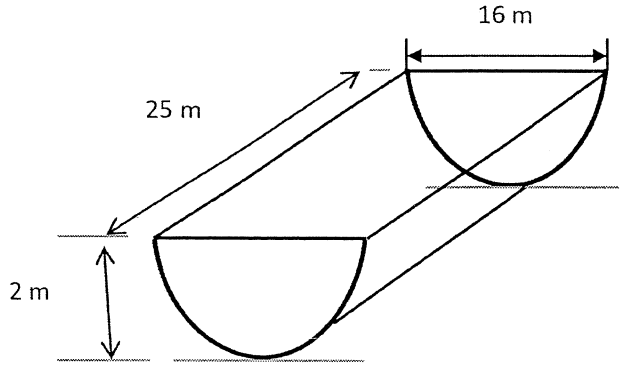

Figure (b): Three dimensional sketch of the tank