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THE OPEN UNIVERSITY OF SRI LANKA
BACHELOR OF INDUSTRIAL STUDIES
TTZ5244 – QUANTITATIVE TECHNIQUES
FINAL EXAMINATION - 2010/2011
DURATION – THREE HOURS

Date: 31st March 2011 **TIME: 1400-1700 Hours**

Answer Question 1 which is compulsory and any other four (04) questions. [Total questions to be answered are 05]. All questions carry 20 marks amounting to a total of 100.

You should clearly show the steps involved in solving problems.

No marks are awarded for mere answers without writing the necessary steps.

Question 1

a. Solve the following equations. **(02 marks)**

(i) $5^{3x+1} \times 25^{x} = 625$ (ii) $3^{4x+1} + 7 = 88$

b. The formula for the accrued sum (S) of an amount A, after n years, when the annual interest (in percentage) rate is r is given by

$$S = A\left(1+\frac{r}{100}\right)^{n}$$

Obtain an expression for n in terms of S, A and r. **(02 marks)**

c. What do you understand by $\left[\frac{dy}{dx}\right]$, if y is a function of x? **(02 Marks)**

d. Differentiate following functions with respect to x **(02 marks)**

(i) $y = 8x^{4} + 5x^{3} + 3x + 7$ (ii) $y = e^{(3x+4)}$

e. Define the "gradient" and the "intercept" of a straight line graph.

(02marks)

f. Determine the stationary points of the following functions and find out whether they are a minima or a maxima. **(04 Marks)**

(a) $y = x^3 - 3x^2 + 1$ (b) $y = 3x^3 - 3x - 2$

g. If $A = \begin{bmatrix} \frac{8}{5} & \frac{3}{7} \end{bmatrix}_{2x2}$ and $B = \begin{bmatrix} 5 & -2 \\ 6 & 4 \end{bmatrix}_{2x2}$ determine $\mathbf{A^2}$ and **AB** **(03 marks)**

h. Find the determinant of the matrix A given below. **(03 marks)**

$$A = \begin{pmatrix} 1 & 0 & 3 \\ 2 & 1 & 4 \\ 1 & 2 & 1 \end{pmatrix}_{3x3}$$

<u>Question 2</u>

a. Describe how straight line graph is used in mathematical modelling.

(04 marks)

b. A statistical model Y = 1000 + 11X postulates to relate X, which is the amount a company spends on advertising in a month to Y, which is the average amount received in sales revenue for the following month.

I. Plot the graph Y vs X.

II. From the graph find out the following

(i) If no money is spent on advertising in a month, what will be the average sales figure in the following month?

(ii) On average, how much increase in revenue would be there in the following month for each Rs. 1/= spent on advertising.

(08 marks)

c. It costs a college a sum of Rs 2000/= for a management student and Rs. 3000/= for a science student. The total budget of the college for the students is Rs. 1,200,000/=. Write an equation to reflect the budget constraints of the college with respect to number of management students and the number of science students. Sketch the graph and mark the feasible region. **(08 marks)**

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Question 3

a. Differentiate the following functions with respect to X

(i) $Y = (6x - 4)(8X^2 + 5X + 4)$

(ii) $Y = \log_e(4X^3 + 2X^2 + 5)$ **(04 Marks)**

b. The "marginal revenue function" is defined as $\frac{dR}{dQ}$, where R is the revenue and Q is the demand. If Q and R are related by the function,

$R = 10Q - 0.001Q^2$,

What is the marginal revenue, when Q = 3000? **(08 Marks)**

c. If the revenue $R = 33Q - 4Q^2$ and

the total cost $C = Q^3 - 9Q^2 + 36Q + 6$

(i). Derive expression for profit in terms of Q **(03 Marks)**

(ii). Determine the output Q, which maximises the profit. **(05 Marks)**

Question 4

Solve the following system of linear equations using matrix 1nversion.

$X + 2Y + 3Z = 4$

$X + 4Y + 2Z = 6$

$2X + Y + Z = 5$

(20 Marks)

Question 5

a. If $A = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}_{3x3}$ show $A^2 - 4A - 5I = 0$, where I is the unit matrix and 0 is the null matrix. **(10 Marks)**

b. If $A=\begin{bmatrix} 1, & 2 \\ -2, & 3 \end{bmatrix}$, $B=\begin{bmatrix} 2, & 1 \\ 2, & 3 \end{bmatrix}$, *And* $C=\begin{bmatrix} -3, & 1 \\ 2, & 0 \end{bmatrix}$

Show that (AB) C = A (BC) and A (B+C) = AB + AC **(10 Marks)**

Question 6

A furniture factory making chairs and tables gets a profit of Rs.200/= per chair and Rs. 300/= per table. Both products are processed on three machines M_1, M_2, and M_3. The time required for processing each product on each of the machines in hours, and the total machine time available in hours per week are given in the table below.

Machine	Processing time in hours		Available machine time in hours / week
	Chair	Table	
M_1	3	3	36
M_2	5	2	50
M_3	2	6	60

a. Name the variables in this problem. **(02 marks)**

b. What are the constraints of the problem? **(04 marks)**

c. Solve the **formatted programme graphically** to determine how the factory should schedule production in order to maximise the profit. **(14 marks)**

Question 7

Use the simplex method to solve the following Linear Programme problem.

Maximise: $Z = 3X_1 + 5X_2 + 4X_3$

Constraints are

$2X_1 + 3X_2 < 8$

$2X_1 + 3X_2 < 10$

$3X_1 + 2X_2 + 4X_3 < 15$

$X_1, X_2, X_3 > 0$ **(20 Marks)**

Question 8

A person wants to decide the constituents of a diet which will fulfil his daily requirements of proteins, fats and carbohydrates at the minimum cost. The choice is to be made from four different types of foods. The yields per unit of these foods are given in the table.

Food type	Yield per unit			Cost per unit
	Proteins	Fats	Carbohydrate	
A	4	3	5	50
B	5	4	3	40
C	10	8	7	80
D	4	5	6	70
Minimum daily requirement	900	300	800	

Formulate a linear programme for the problem.

(i) What are the variables in this problem? **(06 marks)**

(ii) Write the objective of this problem. **(07 marks)**

(iii) What are the constraints of the problem? **(07 marks)**