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The Open University of Sri Lanka

Faculty of Engineering Technology

Industrial Studies (Agriculture) Programme

Final Examination – 2014/ 2015

AEZ3238 Mathematics for Agriculture

Date : 08-08-2015
Time : 9.30-12.30
Duration : Three (03) hours

Read the instruction given below before starting to answer the question paper

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1. This question paper has **eight (08)** questions. Answer any **six (06)** questions.
2. All questions carry equal marks.
3. Answers to questions shall be legible, clear, and neat and step by step procedure is important in all calculations. Final answers should be underlined.
4. Start answering each question from a fresh page with the relevant question number written at the top of the page.
5. It is important that the candidate writes the question numbers of the attempted questions on the cover page.
6. Do not use red ink or pencils to write your answers.
7. Write your Index Number clearly on the answer book.
8. Delete by drawing a line on the page/s in the answer book,where you have material which do not require the attention of the examiner.

Question 1

a) Convert each of the following angle to degree. The value of the angle should be rounded off to the nearest first decimal place and positive value.

i) 1 rad ii) $\frac{3\pi}{2}$ iii) $-\frac{\pi}{2}$

b) Convert each of the following angle to radian:

i) 120^0 ii) 520^0 iii) -270^0

c) Let $\sec\theta = \frac{m^2-1}{2m}$. Find the values of the following trigonometric functions in terms of m:

i) $\cos\theta$
ii) $\sin\theta$
iii) $\tan\theta$

Question 2

i) Prove the following trigonometric relationships

ii) $\cot\theta \sin\theta \sec\theta = 1$

iii) $(1-\cos^2\theta)(1+\tan^2\theta) = \tan^2\theta$

iv) $\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = \text{cosec}\,\theta \sec\theta$

v) $\frac{\tan\theta}{\sec\theta - 1} + \frac{\tan\theta}{\sec\theta + 1} = 2\text{cosec}\,\theta$

a) A ship leaves port **A**at 12.00 noon and sails due west (**W**) at 20 nautical miles per hour. At 2.00 pm the ship is at **B** and changes its course (path) to **N 60^0 W**, as shown in the **Figure1**and sails at the same speed towards port **C**. Find the ship's bearing and distancefrom the port **A**when the ship is at port**C** at 3.00 pm.
[*In usual notations, any triangle with sides**a**, **b** and **c**with opposite angles BAC, CBA and BCAsatisfies the following trigonometric relationships.*]

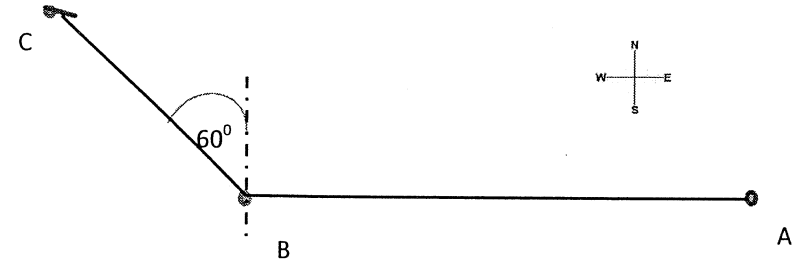

Figure 1

Question 3

a) Factorize the following expressions:

i) $3xy^2 + 12\,x^2y$

ii) $x^2 + 80x - 81$

iii) $n^3 - 4n$

iv) $2x^2 + xy - y^2$

b) Solve the followingequations:

i. $\frac{x-1}{2} = \frac{2x+5}{3}$

ii. $x^2 - 7x + 1 = 0$

iii. $2x^3 - 16 = 0$

c) Solve the following system of equations:

$$x + y = 3$$
$$2x + y - z = 5$$
$$x + 2y + 2z = 3$$

Question 4

a) It is given that the equation $x^2 + (k-3)x + k = 0$has real roots. Find the range of the value of k.

b) In the above equation if the value of $k = +4$, and the roots are given by α and β, find the values of the following:

i) αβ ii) $(\alpha + \beta)$ iii) $(\alpha^2 + \beta^2)$ iv)$(\alpha - \beta)$ v) $\left(\frac{1}{\alpha^2+\beta^2}\right)$

c) Prove that the equation $cx^2 + (c-1)x + 1 - 2c = 0$ has real roots for real values of c

Question 5

a) Evaluate the limits of the following functions:

i) $\lim_{x\to 1} \frac{x^2+3x-4}{x-1}$; ii) $\lim_{x\to 0} \frac{(x+3)^2-9}{x}$ iii) $\lim_{x\to \infty} \frac{x^2+1}{2x^2+x-1}$.

b) Differentiate following functions with respect to the relevant variable.

i) $\frac{2}{x^4}$

ii) $10\,x^{12}$

iii) $x^2+\frac{1}{x}+\sqrt{x}$

iv) $(t^2+t+1)\left(\frac{1}{t}+\sqrt{t}\right)$

v) $\frac{t^2+2t+1}{t-1}$

c) Given that $y = x^6 + 4x^4 + x + 3$. Determine the first and second derivatives of y.

Question 6

a) Let $y = A\cos\omega\theta + B\sin\omega\theta$ where A, B and ω are constants. Show that $\frac{d^2y}{d\theta^2} = -\omega^2\theta$.

b) Suppose that the function $f(x) = \frac{2x}{1+x^2}$ has two minimum turning points and one maximum turning point.

i) What are the coordinates of these turning points?

ii) Check for maxima and minima and identify the nature of the turning points.

Question 7

a) Find the indefinite integral of the following functions.

i) $\int (x+2)^2\, dx$

ii) $\int \left(3x^2 - \frac{1}{x^2} + \frac{1}{x} + \frac{1}{6\sqrt{x}}\right) dx$

iii) $\int x \sin x\, dx$

b) Find the following finite integrals.

i) $\int_0^1 (x^3 + x + 1) dx$

ii) $\int_1^2 \frac{(t+1)}{t^2} dt$

iii) $\int_1^2 2sin2\theta d\theta$

Question 8

The Figure 2(a) below shows the cross section of a reservoir which stores water for irrigation purposes. Figure (b) is a three dimensional sketch of the whole tank. The cross section, which is symmetrical about O-Y axis, is uniform over the tank length. The tank is 20m long and 8 m wide. The profile of the tank wall with respect to x-y coordinate systemwith origin at O is given by the equation, $x^2 = 8y$. The maximum height of the tank is 4m as shown in the Figure2(b). It is required to estimate the volume of the water that the tank can hold.

i. Calculate the cross sectional area of the tank in square meters.*[Hint: Use definite integration and integrate with respect to variable y.]*

ii. Hence find the water holding capacity of the tank in cubic meters.

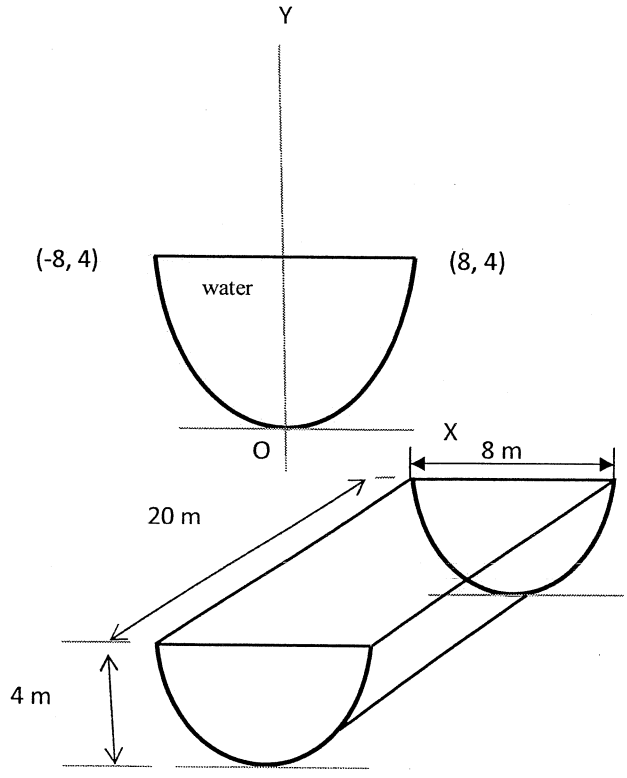

Figure2(a): Sectional profile of the Tank with respect to O-X and O-Y coordinate

Figure2(b): Three dimensional sketch of the tank