Answer guide for
PSE 3117 Mathematics for Chemistry and Biology
2006/2007
Assignment Test I

(1) (a) $(x + 1)^2 - (x + 1)3x - (x^2 - 1)3$

$= x^2 + 2x + 1 - 3x^2 - 3x - 3x^2 + 3$

$= -5x^2 - x + 4$

(b) (i) $2x^2 - 25x + 12 = 0$

$2x^2 - 24x - x + 12 = 0$

$2x(x - 12) - 1(x - 12) = 0$

$(x - 12)(2x - 1) = 0$

$x = 12$ or $x = \frac{1}{2}$

(ii) $\log_{10} 2(4x - 1) - \log_{10}(x^2 + 2x) = 1$

$\log_{10} 2(4x - 1) - \log_{10}(x^2 + 2x) = \log_{10} 10$

$$\log_{10}\left[\frac{2(4x-1)}{x^2 + 2x}\right] = \log_{10}[10]$$

$$\frac{2(4x-1)}{x^2 + 2x} - 10$$

$4x - 1 = 5x^2 + 10x$

$5x^2 + 6x + 1 = 0$

$5x^2 + 5x + x + 1 = 0$

$5x(x+1) + 1(x + 1) = 0$

$(x + 1)(5x + 1) = 0$

$x = -1$ or $x = -1/5$

(iii) 1st Method

$$\frac{2x+1}{2x^2 + 3x - 5} = \frac{A}{(x-1)} + \frac{B}{(2x+5)}$$

$(2x + 1) = A(2x + 5) + B(x - 1)$

When $x = 1$

$2 + 1 = A(2 + 5)$

$A - \frac{3}{7}$

When $x = -5/2$

$$-\left(\frac{5}{2}\right) x\ 2 + 1 = B\left(-\frac{5}{2} - 1\right)$$

$$-5 + 1 - B\left(\frac{-5 - 2}{2}\right)$$

$$-4 = B - \frac{7}{2}$$

$$B = \frac{8}{7}$$

$$\frac{2x+1}{2x^2 + 3x - 5} = \frac{3}{7(x-1)} + \frac{8}{7(2x+5)}$$

2nd Method

$$\frac{2x+1}{2x^2 + 3x - 5} = \frac{A}{(x-1)} + \frac{B}{(2x+5)}$$

$(2x + 1) = A(2x + 5) + B(x - 1)$

When equating

$2 = 2A + B$

$1 = 5A - B$

$1 + 2 \quad 3 = 7A$

$$A = \frac{3}{7}$$

$$2 = 2\left(\frac{3}{7}\right) + B$$

$$2 - \frac{6}{7} = B$$

$$B = \frac{14 - 6}{7}$$

$$B = \frac{8}{7}$$

$$\frac{2x+1}{2x^2 + 3x - 5} = \frac{3}{7(x-1)} + \frac{8}{7(2x+5)}$$

(2) (i)

$$\log_a b = \frac{\log_{10} b}{\log_{10} a}$$

Proof :

Let a and b be any positive real numbers,

$\log_a^{b} = x$; x is a real number

$\Rightarrow a^x = b$

Take the logarithms to the base 10 of both sides, we have,

$$\log_{10}(a^x) = \log_{10} b$$
$$\Rightarrow x\log_{10} a = \log_{10} b$$
$$\Rightarrow x = \log_{10} b / \log_{10} a$$
$$\Rightarrow \log_a b = \frac{\log_{10} b}{\log_{10} a}$$

(ii)

$$\left[\frac{\log_{10} 125}{\log_{10} 25}\right] \log_3 5 + \log_3 50 + \log_{10} 0.03$$
$$= \left[\frac{\log_{10} 5^3}{\log_{10} 5^2}\right] \frac{\log_{10} 5}{\log_{10} 3} + \frac{\log_{10} 5 \text{ x } 10}{\log_{10} 3} + \log_{10} 3/10^2$$
$$= \left[\frac{3 \log_{10} 5}{2 \log_{10} 5}\right] \frac{\log_{10} 5}{\log_{10} 3} + \frac{\log_{10} 5 + \log_{10} 10}{\log_{10} 3} + \log_{10} 3 - 2\log_{10} 10$$
$$= \frac{3}{2} \text{ x } \frac{0.6990}{0.4771} + \frac{(0.6990+1)}{0.4771} + 0.4771 - 2$$
$$= \frac{3}{2} \text{ x } \frac{0.6990}{0.4771} + \frac{0.6990}{0.4771} + \frac{1}{0.4771} + 0.4771 - 2$$
$$= \frac{5}{2} \text{ x } \frac{0.6990}{0.4771} + \frac{1}{0.4771} + 0.4771 - 2$$
$$= 3.6627 + 2.0959 + 0.4771 - 2$$
$$= 4.2357$$

(3) (a)

$$\ln y - 3\ln x + 2$$
$$= \ln x^3 + 2$$
$$\ln y - \ln x^3 = 2$$
$$\ln \frac{y}{x^3} = 2$$
$$\frac{y}{x^3} - e^2$$
$$y = e^2 x^3$$

(b)

$$\frac{5(1-i)(3-i)}{(3+i)(3-i)} - (1-2i) + i^3$$
$$= \frac{5(3 - 4i + i^2)}{9 - i^2} - 1 + 2i - i$$
$$= \frac{5(2-4i)}{10} - 1 + 2i - i$$
$$= 1 - 2i - 1 + 2i - i$$
$$= -i$$

(4) (i)

$\sin^3\theta = \frac{1}{4}(3\sin\theta - \sin 3\theta)$

R.H.S. $= \frac{1}{4}[3\sin\theta - \sin 3\theta]$

$= \frac{1}{4}[3\sin\theta - \sin(\theta + 2\theta)]$

$= \frac{1}{4}[3\sin\theta - (\sin\theta\cos 2\theta + \cos\theta\sin 2\theta)]$

$= \frac{1}{4}[3\sin\theta - \sin\theta(1 - 2\sin^2\theta) - \cos\theta 2\sin\theta\cos\theta]$

$= \frac{1}{4}[3\sin\theta - \sin\theta + 2\sin^3\theta - 2\sin\theta(1 - \sin^2\theta)]$

$= \frac{1}{4}[3\sin\theta - 3\sin\theta + 4\sin^3\theta]$

$= \frac{1}{4}4\sin^3\theta$

$= \sin^3\theta$

= L.H.S.

(ii)

$\cos 2\theta + 3\sin\theta - 2 = 0$

$1 - 2\sin^2\theta + 3\sin\theta - 2 = 0$

$2\sin^2\theta - 3\sin\theta + 1 = 0$

$(\sin\theta - 1)(2\sin\theta - 1) = 0$

$\sin\theta = 1$ or $\sin\theta = ½$

$\theta = \pi/2$ or $\theta = \pi/6, 5\pi/6$

(iii) $(\sin\theta + \cos\theta)^2$

$= \sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta$

$- \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta$

$= 1 + \sin 2\theta$

$- 1 + p$

$\therefore (\sin\theta + \cos\theta)2 = 1 + p$

$\sin\theta + \cos\theta = \pm\sqrt{1 + p}$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$q - (\cos\theta + \sin\theta)(\cos\theta - \sin\theta)$

$q = \pm\sqrt{1 + p}\,(\cos\theta - \sin\theta)$

$\therefore (\cos\theta \quad \sin\theta) = \pm\frac{q}{\sqrt{1 + p}}$

(5) (a) $= \lim_{x \to 3} \frac{2x^2 - 7x + 3}{x^2 - 9}$

$= \lim_{x \to 3} \frac{(x - 3)(2x - 1)}{(x - 3)(x + 3)}$

$= \lim_{x \to 3} \frac{(2x - 1)}{(x + 3)}$

$= \frac{(2 \times 3 - 1)}{(3 + 3)}$

$= \frac{5}{6}$

(b)

$y = \frac{1}{x}$ ----------(1)

$y + \delta y = \frac{1}{x + \delta x}$ ---------- (2)

(2) – (1) $\delta y = \frac{1}{x + \delta x} - \frac{1}{x}$

$\delta y - \frac{x - (x + \delta x)}{(x + \delta x)x}$

$\delta y = \frac{x - x - \delta x}{(x + \delta x)x}$

$\delta y = \frac{-\delta x}{(x + \delta x)x}$

Both sides devided by δx

$\frac{\delta y}{\delta x} = \frac{-\delta x}{(x + \delta x)x.\delta x}$

$\lim_{\delta x \to 0} \frac{\delta y}{\delta x} = \frac{-1}{(x + \delta x)x}$

$\frac{dy}{dx} = \frac{-1}{x^2}$

(6) (i)

$$u = (2-3x)^9$$
$$\frac{du}{dx} = 9(2-3x)^8\,(-3)$$
$$= -27(2-3x)^8$$

(ii)

$$P = RT^2(3-a)$$
$$\frac{dP}{dT} = 2RT(3-a)$$

(iii)

$$V = \frac{(u+2)}{(u^2-1)}$$
$$\frac{dV}{dU} = \frac{(u^2-1)\frac{d(u+2)}{du} - (u+2)\frac{d(u^2-1)}{du}}{(u^2-1)^2}$$
$$= \frac{(u^2-1)1 - (u+2)(2u)}{(u^2-1)^2}$$
$$= \frac{u^2-1-2u^2-4u}{(u^2-1)^2}$$
$$= \frac{-u^2-4u-1}{(u^2-1)^2}$$

(iv)

$$y = \mathrm{Sin}2x.\ln x$$
$$\frac{dy}{dx} = \mathrm{Sin}2x.\frac{1}{x} + \ln x.\mathrm{Cox}2x.2$$
$$= \frac{\mathrm{Sin}2x}{x} + 2\ln x.\mathrm{Cox}2x$$

(7)

$$\psi = (x+1)e^x$$
$$\frac{d\psi}{dx} = (x+1)e^x + e^x$$
$$= (x+2)e^x$$
$$\frac{d^2\psi}{dx^2} = (x+2)e^x + e^x$$
$$= (x+3)e^x$$
$$\frac{d^3\psi}{dx^3} = (x+3)e^x + e^x$$
$$= (x+4)e^x$$

$$\frac{d^3\psi}{dx^3} + \psi - \frac{d\psi}{dx} - \frac{d^2\psi}{dx^2}$$
$$= (x+4)e^x + (x+1)e^x - (x+2)e^x - (x+3)e^x$$
$$= 0$$

(8)

$$\left(P - \frac{2}{V^2}\right)(V+2) = 2T$$
$$P - \frac{2}{V^2} = \frac{2T}{(V+2)}$$
$$P = \frac{2T}{(V+2)} + \frac{2}{V^2}$$

$$\frac{\partial P}{\partial T} = \frac{2}{V+2}$$

$$\frac{\partial P}{\partial V} - \frac{(V+2)\frac{\partial\, 2T}{\partial V} - 2T\frac{\partial (V+2)}{\partial V}}{(V+2)^2} + \frac{\partial\left(2/V^2\right)}{\partial V}$$
$$= -\frac{2T}{(V+2)^2} - \frac{4}{V^3}$$

$$dP = \left(\frac{\partial P}{\partial T}\right)_V dT + \left(\frac{\partial P}{\partial V}\right)_T dV$$
$$= \frac{2}{V+2}dT - \left(\frac{2T}{(V+2)^2} + \frac{4}{V^3}\right)dV$$

$$\left\{\frac{\partial}{\partial T}\left(\frac{\partial P}{\partial V}\right)_T\right\}_V = -\frac{2}{(V+2)^2}$$
$$\left\{\frac{\partial}{\partial V}\left(\frac{\partial P}{\partial T}\right)_V\right\}_T = -\frac{2}{(V+2)^2}$$
$$\left\{\frac{\partial}{\partial T}\left(\frac{\partial P}{\partial V}\right)_T\right\}_V = \left\{\frac{\partial}{\partial V}\left(\frac{\partial P}{\partial T}\right)_V\right\}_T$$