00188

THE OPEN UNIVERSITY OF SRI LANKA
BACHELOR OF TECHNOLOGY / BACHELOR OF SOFTWARE ENGINEERING – LEVEL 04
FINAL EXAMINATION – 2014/2015
MPZ4140 / MPZ4160 - DISCRETE MATHEMATICS I
DURATION – THREE HOURS

Date: 06th September 2015 **Time: 0930hrs – 1230 hrs.**

Instructions:

- **Answer only six questions.**
- **Please answer a total of six questions choosing at least one from each section A, B, and C.**
- **State any assumption that you made.**
- **All symbols are in standard notation.**

SECTION – A

01. i. Decide which of the following are propositions: **[20%]**
   a) "Sum of an even and odd integer is an even integer".
   b) "He has constructed a beautiful house".
   c) "4+4 = 7 and 4 – 3 = 5".
   d) "Are all circles round?"

ii. Given the propositions:
   p : Question paper are easy
   q : We pass the examination.
   r : The principal declares a holidays.
   s : We are happy.

Express each of the following in symbolic form:

a) If the question papers are easy or we are happy then the principal declares a holiday.
b) If the question papers were not easy, then we do not pass the examination and the principal do not declare a holiday". **[20%]**

iii. Let p, q, and r be three statements.
Verify that $[(p \vee q) \rightarrow r] \leftrightarrow [\sim r \rightarrow \sim(p \vee q)]$ is a tautology or not.
[20%]

iv. State the "converse", "inverse", and "contrapositive" of each of the following statement: **[30%]**

a) If x is less than zero then x is not a prime number.
b) If I have time and I am not tiered then I will go for a work.

v. Show that $p \wedge (p \vee q) \equiv p$, by using laws of the algebra of propositions. **[10%]**

02. i. Test the validity of the following argument:

If there was a cricket match, then travelling was difficult.
If they arrived on time then traveling was not difficult.
They arrived on time.
Therefore there was no cricket match.

[30%]

ii. Prove each of the following statement is true. **[20%]**

a) $\exists x \in \mathbb{R},\ x^3 + x^2 - 2 = 0,$
b) $\exists r \in \mathbb{Q},\ \ \sin(\pi r) = \frac{1}{2}.$

iii. Give the negation of the following statements:

a) $\exists x \in A,\ x + 3 = 10,$
b) $\exists x \in A,\ x + 3 \leq 7,$
c) $\forall x, \forall y,\ p(x, y).$ **[30%]**

iv. Prove distributive's laws by using truth tables. **[20%]**

03. i. Using mathematical induction, for a positive integer *n*, prove each of the following:

a) $f(n) = 6^n$-5n+4 is divisible by 5.
b) $\sum_{r=1}^{n} r(r+1) = \frac{1}{3} n(n+1)(n+2).$

[50%]

ii. Prove that, if $x + y$ is an irrational number, then x is an irrational number or y is an irrational number. **[30%]**

iv. Show that $\sqrt{5}$ is an irrational number. **[20%]**

SECTION – B

04. i. Find the all elements in each of the following set: **[15%]**

$A = \{x : x = 2n + 1,\ n \in \mathbb{N},\ n < 11\},$
$B = \{x : x = n^3 + n^2, 0 \leq n \leq 5, n \in \mathbb{Z}\},$
$C = \{x : x = 1 + (-1)^n,\ n \in \mathbb{Z}\}.$

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ii. Explain the difference between $A \subseteq B$ and $A \subset B$. . **[10%]**

iii. Without using Venn diagram, show that
$(A \cup B)' = A' \cap B'$. **[20%]**

iv.

a) Let $|A \cup B| = |A| + |B| - |A \cap B|$. Show that
$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$. **[20%]**

b) In a class consisting of 120 students, 30 are studying ***C***, 40 are studying ***Pascal*** and 45 are studying ***Java***, 15 are studying both ***C*** and ***Pascal***, 20 studying both ***Pascal*** and ***Java***, 12 studying both ***C*** and ***Java***, 8 are studying all the three. Without using Venn diagram answer the following,
how many do not take any of these subjects?
how many take only one language? **[35%]**

05. i. Define the cartesian product sets of A and B.
Let $A = \{a, ab, b\}$, and $B = \{n, nm, m\}$. Find $A \times B,\ \ A^2 \times B$. **[25%]**

ii. A binary operation $f: \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$, is defined by $f(a, b) = a \cdot |b|,\ \forall a, b \in \mathbb{Z}$.
Show that f is not commutative but associative. **[25%]**

iii. Prove that the following relation is equivalence relation and describe the equivalence classes.
The relation $pRq \Leftrightarrow p^2 + q^2$ is divisible by 2 on the set $\mathbb{Z}$. **[25%]**

iv. Prove that the following relation on the set of integer is a partial order. **[25%]**

$$R_1 = \{(x, y): x \leq y\}.$$

06. i. Define a function from set A into a set B. **[10%]**

ii. Let $A = \mathbb{R} - \{-4\}$ and $B = \mathbb{R} - \{1\}$. Define $f: A \to B$ by $f(x) = \frac{x+5}{x+4}$. Prove that f is invertible and find a formula for f^{-1}. **[40%]**

iii. Let $g: \mathbb{R} \to \mathbb{R}$ be defined by

$$g(x) = \begin{cases} x + 7\ ; & x \leq 0 \\ -2x + 5; & 0 < x < 3\ . \\ x - 1; & x \geq 3 \end{cases}$$

Find $g(0), g(-10), g(2)$, and $g(3)$. **[20%]**

iv. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ and $g: \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = ax + b$ and $g(x) = cx + d$ respectively for all $x \in \mathbb{R}$, where $a, b, c,$ and d are constants. Find the relationship(s) between the constant $a, b, c, d,$ $if\ f \circ g(x) = g \circ f(x)$ for all x. **[30%]**

SECTION – C

07. i. Let $a, b,$ and c be any integer numbers. Prove that,

a) If $a|b$ and $b|a$, then $a = \pm b$. **[20%]**

b) If $31|(5a+7b+11c)$, then $31|(21a + 17b + 9c)$. **[25%]**

ii. If $n \in \mathbb{Z}^+$ and n is odd, prove that $8|(n^2 - 1)$. **[20%]**

iii. Let $a, b \in \mathbb{Z}^+$. If $b|a$ and $b|(a+2)$, prove that $b = 1$ or 2. **[15%]**

iv Prove that If $a|b$ and $a|c$,then $a|(mb+nc)$ for any integers m and n. **[20%]**

08. i. Let $a, b \in \mathbb{Z}^+$and $a \geq b$. Prove that $cd(a,b) = gcd(a-b, b)$. **[10%]**

ii. Let a, b and c are three integers such that $gcd(a,c) = 1$ and $gcd(b,c) = 1$. Show that $gcd(ab, c) = 1$. **[15%]**

iii. Let a and b are integers and $gcd(a,b) = 1$. Prove that $cd(ac, b) = gcd(c, b)$, where $\in \mathbb{Z}$. **[25%]**

iii. Find the $gcd(275,726)$, and express it as $275x + 726y$ by using the Euclidean Algorithm.
Determine all integer solutions of the following Diophantine equation:
$275x + 726y = 27500$.
[50%]

09.

i. Let $a \equiv b(mod\ m)$ and $c \equiv d(mod\ m)$. Show that $ac \equiv bd(mod\ m)$. **[15%]**

ii. Solve each of the following system of congruence:

a)
$x \equiv 1(mod\ 2)$
$x \equiv 2(mod\ 3)$
$x \equiv 3(mod\ 5)$
$x \equiv 1(mod\ 7)$

b)
$x + 5 \equiv 7(mod\ 5)$
$x - 1 \equiv 2(mod\ 6)$
$x + 2 \equiv 4(mod\ 7)$

[85%]