The Open University of Sri Lanka
Faculty of Engineering Technology

00101

Study Programme	:	Bachelor of Technology (Engineering)
Name of the Examination	:	Final Examination
Course Code and Title	**:**	**MEX4272 Vibration and Fault Diagnosis**
Academic Year	:	2014/15
Date	:	August 07, 2015
Time	:	0930 hrs. – 1230 hrs.
Duration	:	3 hours

General instructions

1. Read all instructions carefully before answering the questions.
2. This question paper consists of 8 questions. All questions carry equal marks.
3. Answer any 5 questions only.

Question 1

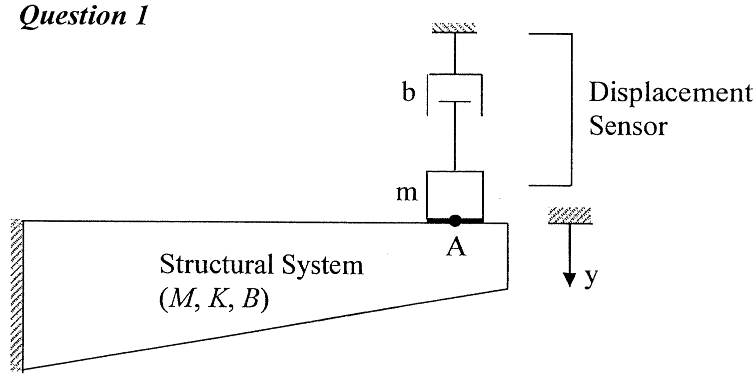

Fig. Q1-a

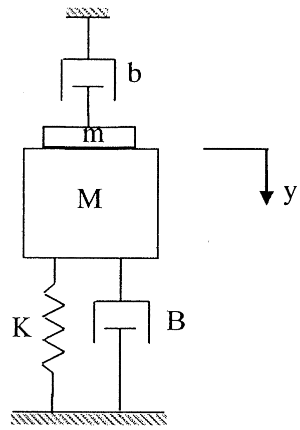

Fig. Q1-b

A vibrating system has an effective mass M, an effective stiffness K, and an effective damping constant B in its primary mode of vibration at point A with respect to coordinate y. Write expressions for the undamped natural frequency, the damped natural frequency, and the damping ratio for this first mode of vibration of the system. A displacement transducer is used to measure the fundamental undamped natural frequency and the damping ratio of the system by subjecting the system to an initial excitation and recording the displacement trace at a suitable location (point A along y in Fig Q1-a) in the system. This trace will provide the period of damped oscillations and the logarithmic decrement of the exponential delay from which the required parameters can be computed using well known relations. It was found, however, that the mass m of the moving part of the displacement sensor and the associated equivalent viscous damping constant b are not negligible. Using the model shown in in Fig. Q1-b, derive expressions for the measured undamped natural frequency and damping ratio.

Question 2

Fig. Q2 shows an inverted pendulum consisting of a spherical ball of radius R and mass m_2 and a uniform rod of mass m_1 and length a. Pendulum is pivoted at O. Two springs, each of stiffness k, are attached to the ball as shown in the figure. Determine the equation of motion and derive an expression for the natural frequency of vibrations of the system. Take the moment of inertia of a sphere about its centre as $\frac{2}{5}mr^2$, where m is the mass and r is the radius of the sphere.

If, $m_1 = m$, $m_2 = \frac{m}{5}$ and $R = \frac{a}{4}$, show that the natural frequency ω_n is approximately given by,

$$\omega_n = 0.438\sqrt{25\frac{k}{m} - 6\frac{g}{a}}.$$

What is the condition for stability of the system?

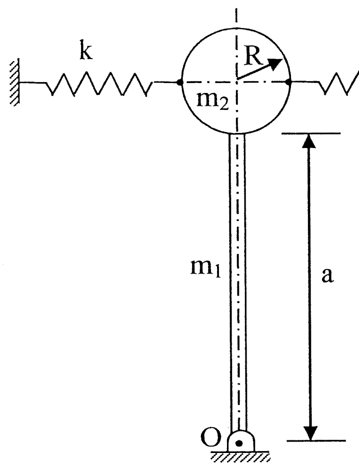

Fig. Q2

Question 3

Fig. Q3 shows two torsional systems. System A consists of two rotors each of moment of inertia I and three shaft sections each having a torsional stiffnesses K while the System B consists of a single rotor of moment of inertia J and two shaft sections of torsional stiffnesses $2K$ and $4K$.

Show that, if $J = 6I$ the lower natural frequency of torsional oscillations of system A is equal to that of system B.

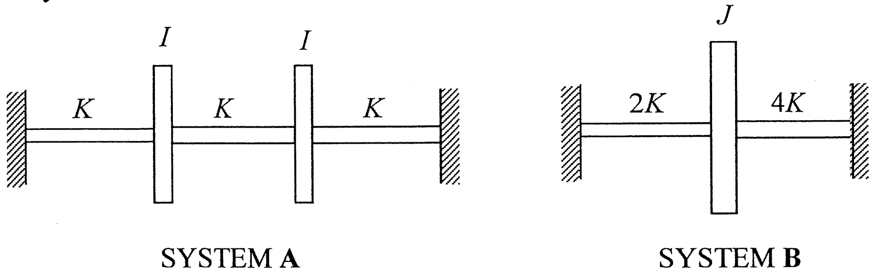

Fig. Q3

Question 4

Fig. Q4 shows a double pendulum (or a two link robot with revolute joints) having arm lengths L_1 & L_2 and end masses m_1 & m_2.

(a) Obtain the equations of motion for the system, using Lagrange's equation in terms of the absolute angles of swing θ_1 and θ_2 about the vertical. Linearise the equations for small motions of θ_1 and θ_2.

(b) For the special case of $m_1 = m_2 = m$ and $L_1 = L_2 = L$, solve the modal problem of this system and obtain natural frequencies in terms of ω_o where $\omega_o^2 = g/L$.

(c) Determine the corresponding modal masses and modal stiffnesses and show that the natural frequencies can be obtained from the modal masses and modal stiffnesses.

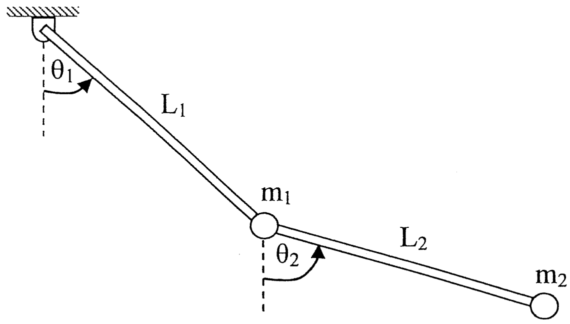

Fig. Q4

Question 5

Fig. Q5 shows a spring-mass system consisting of two masses m_1 and m_2 and four springs of stiffnesses k_1, k_2, k_2 and k_3.

(a) Derive the equations of motion and express them in the form of a matrix.

(b) Obtain the frequency equation (You do not need to simplify this equation).

(c) If, $k_1 = k_2 = k$, $k_3 = 2k$, $m_1 = m$ and $m_2 = 3m$,

(i) show that the frequency equation can be expressed as

$$\omega^4 - \frac{10}{3}\left(\frac{k}{m}\right)\omega^2 + \frac{7}{3}\left(\frac{k}{m}\right)^2 = 0,$$

where ω is the natural frequency of vibration.

(ii) determine the natural frequencies of vibration of the system in terms of k and m and the ratio of amplitudes of motion of the two masses corresponding to each frequency.

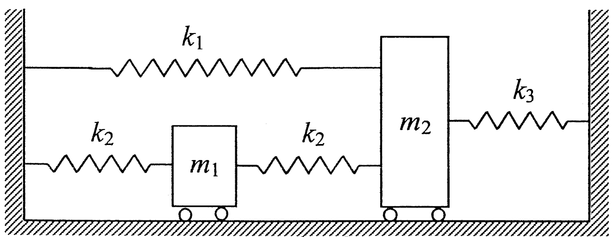

Fig. Q5

Question 6

(a) Fig. Q6(a) shows a unity feedback control system. The unit step response of the system which was initially at rest is $c(t) = 1 - e^{-3t}(\cos 2t + 1.5\sin 2t)$. Determine the closed loop transfer function of the system and the transfer function $G(s)$.

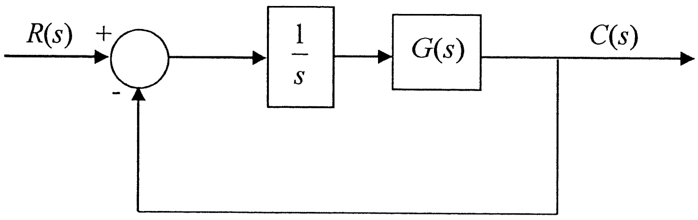

Fig. Q6-a

(b) Find the Fourier transform of the rectangular pulse signal $x(t)$ defined by

$x(t) = \begin{cases} 1 & -a < t < a \\ 0 & t < -a, t > a \end{cases}$ and shown in Fig. Q6(b). Sketch the Fourier transform.

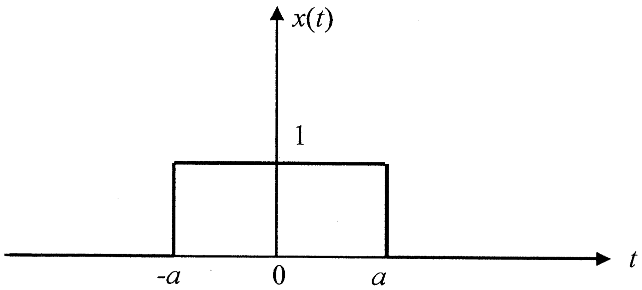

Fig. Q6-b

Question 7

A dynamic model of a fluid coupling system is shown in Fig. Q7. The fluid coupler is represented by a rotary viscous damper with damping constant b. It is connected to a rotary load of moment of inertia I restrained by a torsional spring of stiffness K as shown. Obtain the frequency transfer function of the system relating the restraining torque T of the spring (output) to the angular displacement excitation $\theta(t)$ (input) that is applied to the far end of the fluid coupler. If $\theta(t) = \theta_o \sin \omega t$, what is the magnitude of T at steady state.

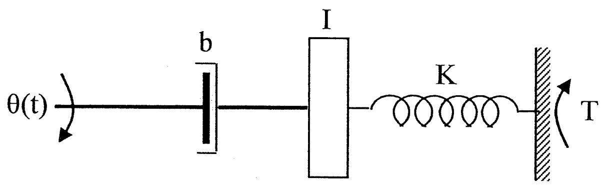

Fig. Q7

Question 8

(a) Maintenance strategies can be broadly classified under three main types. Write a detailed account of these three types.

(b) Define the terms, Mechanical Impedance; Mobility; Force Transmissibility; and Motion Transmissibility.

(c) In a vibration test, a harmonic forcing excitation is applied to the test object by means of a shaker and the velocity response at some other location is measured by using an accelerometer.

i. State whether the above mentioned test is a mobility test or an impedance test. Justify your answer.

ii. Explain how the velocity response can be measured using an accelerometer.

LAPLACE TRANSFORMS

TIME FUNCTION $f(t)$	LAPLACE TRANSFORM $F(s)$
Unit Impulse $\delta(t)$	1
Unit step	$\frac{1}{s}$
t	$\frac{1}{s^2}$
t^n	$\frac{n!}{s^{n+1}}$
$\frac{df(t)}{dt}$	$sF(s)-f(0)$
$\frac{d^n f(t)}{dt^n}$	$s^n F(s) - s^{n-1} f(0) - s^{n-2} \frac{df(0)}{dt} \ldots - \frac{d^{n-1} f(0)}{dt^{n-1}}$
e^{-at}	$\frac{1}{s+a}$
te^{-at}	$\frac{1}{(s+a)^2}$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
$e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at} \cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$