THE OPEN UNIVERSITY OF SRI LANKA

BACHELOR OF TECHNOLOGY HONORS IN ENGINEERING – LEVEL 03

FINAL EXAMINATION - 2015/2016

MPZ 3132 – ENGINEERING MATHEMATICS IB

DURARTION: THREE HOURS

Date: 21st November 2016 **Time: 0930hrs - 1230hrs**

Instructions:

- **Answer five (05) questions only.**
- **Number of pages in the paper – 05.**
- **All symbols are in standard notation.**

Important integrals

- $\int e^{ax} sinbxdx = \frac{e^{ax}}{a^2+b^2}(asinbx - bcosbx)$
- $\int e^{ax} cosbxdx = \frac{e^{ax}}{a^2+b^2}(acosbx + bsinbx)$
- $\int f(x)e^{ax}\,dx = \frac{1}{a}f(x)e^{ax} - \frac{1}{a^2}f^1(x)e^{ax} + \frac{1}{a^3}f^2(x)e^{ax}$ …… [Sign alternate $(+-+-+\cdots)$]
- $\int f(x)cosax\,dx = \frac{1}{a}f(x)sinax + \frac{1}{a^2}f^1(x)cosax - \frac{1}{a^3}f^2(x)sinax - \cdots$

  [Sign alternate in pairs $++\ --++--++\cdots$)]
- $\int f(x)sinax\,dx = -\frac{1}{a}f(x)cosax + \frac{1}{a^2}f^1(x)sinax + \frac{1}{a^3}f^2(x)cosax - \cdots$

  [Sign alternate in pairs after the first term $(++--++--++--\cdots)$]
- $\int secax\,dx = \frac{1}{a}ln|secax + tanax|$

1)

I. Write down the Fourier series expansion function $f(x)$ with period $2l$. **[15%]**

II. Find the Fourier series of $f(x) = x^2,\ 0 < x < 2\pi$ with period 2π **[25%]**

III. Find the Fourier series representation of the function with period 2π, defined by

$f(x) = 200ax + 300bx^2 + 400cx^3$ for $-\pi < x < \pi$.

Hence deduces that $1 + \frac{1}{4} + \frac{1}{9} + \cdots = \frac{\pi^2}{6}$. **[60%]**

2)

I. Define the n[th] order Taylor polynomial of $f(x)$ about $x = a$. **[05%]**

a) Using the Taylor series expansion prove that $ln(1-x) = -\sum_{r=0}^{\infty} \frac{x^{r+1}}{r+1}$ where $|x| < 1$. **[20%]**

b) Using the above formula prove that $ln\left(\frac{a+x}{a-x}\right) = \sum_{r=1}^{\infty} \frac{(1-(-1)^r)x^r}{a^r\, r}$ **[15%]**

II. If $y = \ln(a + b\cos\theta)$, then show that

$$\frac{d^3y}{d\theta^3} + \frac{dy}{d\theta}\left(1 + 3\frac{d^2y}{d\theta^2}\right) + \left(\frac{dy}{d\theta}\right)^3 = 0.$$

Find the third order Taylor polynomial about $\theta = 0$. **[60%]**

3)

I. Solve the following differential equation:

$$(D^4 - 81)y = 0$$ **[15%]**

II. Using the formula $\frac{1}{D+\alpha} f(x) = e^{-\alpha x}\frac{1}{D} e^{\alpha x} f(x)$,

find the particular integral of the following differential equation

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 5e^{-3x}.$$

Hence find the general solution of the above differential equation. **[35%]**

III. Using a suitable trial function,

find a particular integral for the following differential equation:

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 25y = 2\sin(x/2) - \cos(x/2).$$

Find the general solution of the above differential equation. **[50%]**

4)

I. Define the "Laplace transformation" of $F(t)$.

Find the "Laplace transformation" of each of the following functions:

a) $5t^2 - 3t - 24e^{-3t} + 32\cos 5t$

b) $f(t) = \begin{cases} e^t & x \leq 2 \\ 3 & x > 2. \end{cases}$ **[35%]**

II. Using the shift theorem, find the "Laplace transformation" of the following function:

$\mathcal{L}\{e^{-5t}(t + 5\sin 3t - 3\cos 2t)\}$ **[15%]**

III. Find the inverse "Laplace transformation" of the following: **[30%]**

$$\frac{8}{s^3(s^2-s-2)}.$$

IV. Using the "Laplace transformation", solve the following differential equation:

$y'' + 9y = 0, \quad y(0) = 3, \quad y'(0) = -5.$ **[20%]**

5)

I. Define the Poisson distribution and estimate its expectation and variance. **[40%]**

Births in a hospital occur randomly at an average rate of 1.8 births per hour.

(a) What is the probability of observing 4 births in a given hour at the hospital? **[10%]**

(b) What about the observing more than or equal to 2 births in a given hour at the hospital? **[15%]**

II. The scoring of modern IQ tests is such that Intelligence Quotients (IQs) have Normal distribution with $\mu = 100$ and $\sigma = 15$.

(a) What percent of people have IQ less than 80? **[10%]**

(b) What percent of people have IQ greater than 120? **[10%]**

(c) International College is a non-profit organization that accepts only people with IQ within the top 2%. What level of IQ qualifies one to be a member of College? **[15%]**

6)

I.

a) Express $\log(1 - i)$ in the form of $x + iy$ where x and y are real numbers. **[15%]**

b) Let $f(Z) = Z^3$, $|Z| \leq 2$ and $0 \leq \arg(Z) \leq \pi/3$. Find the image of the function f in W-plane and draw the image of the function in Z and W planes. **[20%]**

c) Determine the general value of the e^Z, z for which have real values. **[15%]**

II.

a) If $\underline{a}$ and $\underline{b}$ are constant vectors, find $\frac{d|t\underline{a} + t^2\underline{b}|}{dt}$. **[20%]**

b) A particle P moves with velocity $3\underline{i} - 4\underline{j}$ from point (3, -2). At the same instant a particle Q, moving in the same plane with velocity $2\underline{i} - \underline{j}$, passes through a point (4, -5). Find whether P and Q collide or not. **[30%]**

7)

I. Find the moment of inertia of the following bodies about the given axis.

a) A uniform circular disc of mass M and radius a, the axis perpendicular to the plane of the disc passing through the center. **[20%]**

b) A uniform solid right circular cone of mass M, height h and radius a about its axis. **[35%]**

II. A Uniform rod of length $2a$ and mass M is free to rotate in a vertical plane about a horizontal axis through one end P of the rod. When the other end Q of the rod is hanging vertically below P it is given an angular velocity $\sqrt{\frac{7g}{a}}$. The rod next comes to rest when PQ is horizontal. Find the moment of the constant frictional couple acting at the axes. (moment of inertia of the uniform rod passing through the midpoint of the rod is $\frac{1}{3}Ma^2$) **[45%]**

8).

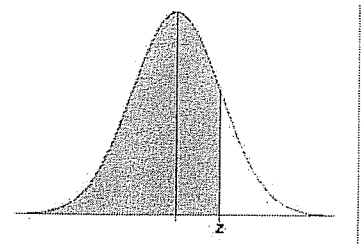
I. A rectangle is immersed vertically in water with two sides horizontal and at depth a and $a + b$ respectively below the effective surface. Prove that the distance of the center of pressure from the upper side is $\frac{b(3a+2b)}{3(2a+b)}$.

Show also that the center pressure is below the center of gravity of the area, and as the depth increase, approaches but never coincides with it. **[60%]**

II. Prove that the depth of the center of pressure of a parallelogram two of whose sides are horizontal and at depths a, $3a$ below the surface of a liquid, whose density varies as the depth below the surface is $\frac{30a}{13}$. **[40%]**

END

Standard Normal Cumulative Probability Table



Cumulative probabilities for POSITIVE z-values are shown in the following table:

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998