00640

THE OPEN UNIVERSITY OF SRI LANKA

Final Examination 2015/2016

Bachelor of Technology Honours in Engineering (Level 3)

MPZ3231-Engineering Mathematics IA

Duration: Three Hours

Date: 29-11-2016 | Time : 0930 hrs - 1230 hrs

Instructions

- Answer **Five (05) questions only**, selecting **at least two (02) questions from each section A and B.**
- This is a closed book Examination.
- This paper contains 5 pages.
- Show all your workings.
- Bold letters represent the vectors.
- All symbols are in standard notations.

SECTION A

1. (a) If $A=\begin{pmatrix} 3 & 2 & 0 \\ -3 & -3 & -1 \\ 4 & 4 & 1 \end{pmatrix}$

   i. Find A^2, A^3 and A^4.

   ii. Deduce that the inverse matrix of A.

iii. Hence find the solutions of the system

$$3x + 2y = 12$$
$$3x + 3y + z = 16$$
$$4x + 4y + z = 21$$

[50%]

(b) Show that $\begin{bmatrix} 1/3 & -2/3 & 2/3 \\ -2/3 & 1/3 & 2/3 \\ -2/3 & -2/3 & -1/3 \end{bmatrix}$ is an orthogonal matrix. [20%]

(c) Show that $\Delta = \begin{vmatrix} 1 & a & a^3 \\ 1 & b & b^3 \\ 1 & c & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$ [30%]

2. (a) Let $y = f(x) = \dfrac{(x+1)(x-2)}{x^2}$

i. Find the horizontal, vertical and slant asymptotes of the curve $y = f(x)$. [15%]

ii. Find $\dfrac{dy}{dx}$, and determine the sign of $\dfrac{dy}{dx}$ as x varies from $-\infty$ to $+\infty$. [15%]

A. Find the turning points of the curve. [15%]

B. Sketch the graph of the function $f(x) = \dfrac{(x+1)(x-2)}{x^2}$ [15%]

C. Deduce the graph of the function $y = g(x) = \dfrac{|(x+1)(x-2)|}{x^2}$ [10%]

(b) If $z = f(x, y)$, $x = uv$, $y = \dfrac{(u^2 - v^2)}{2}$, then show that

$u\dfrac{\partial z}{\partial v} - v\dfrac{\partial z}{\partial u} = 2[y\dfrac{\partial z}{\partial x} - x\dfrac{\partial z}{\partial y}]$ [30%]

3. (a) i. Express the complex number $z = (-1 + i)$ in the form $r\{\cos\theta + i\sin\theta\}$ where $r > 0$ and $-\pi \leq \theta \leq \pi$ [10%]

ii. Prove that $|(1+i)^n + (1-i)^n| = 2\{2^{\frac{n}{2}} \cos\dfrac{n\pi}{4}\}$ [30%]

(b) Sketch the locus of z and find the cartesian equation of the locus.

i. $|z+3i| = |z-5|$ [15%]

ii. $\text{Arg}\,[z-(2-3i)] = \frac{\pi}{6}$ [10%]

$\text{Arg}\,[z-(2-3i)] = \frac{7\pi}{6}$ [10%]

iii. $|z+5i| = 5$ [10%]

iv. $|z-2i|^2 - |z+2i|^2 = 10$ [15%]

4. (a) In a $OABC$ tetrahedron, the edges OA,OB, and OC are mutually perpendicular. $OA = OB = OC = \sqrt{2}a$.

Find

i. the lengths of AB, BC, and CA. [05%]

ii. the angle between the planes ABC and OBC. [15%]

iii. the area of the triangle ABC. [05%]

iv. the perpendicular distance to the plane ABC from the point O. [10%]

v. volume of the tetrahedron. [15%]

(b) Find the coordinates of point, where the line $\frac{x-5}{1} = \frac{y-3}{2} = \frac{z-3}{2}$ meets the plane $2x+y+z = 11$ [30%]

(c) Find the equation of the normal to the parabola $y^2 = 4ax$ at the point $P \equiv (at^2, 2at)$ [20%]

SECTION B

5. (a) Solve the differential equations.

i. $\frac{dy}{dx} = x^3 e^{-y}$. [15%]

ii. $(x+1)\frac{dy}{dx} + 1 = 2e^{-y}$. [25%]

(b) Solve the following differential equation by using an integral factor, $\frac{dy}{dx} + \frac{y}{x} = \frac{\cos x}{x}$. [30%]

(c) Solve $(x^3 - 3xy^2)dx + (y^3 - 3x^2y)dy = 0$ when $y = 1$ at $x = 0$. [30%]

6. (a) Given that the position vectors of the point A, B, C and R are $\mathbf{a} = 2\mathbf{i} + 2\mathbf{j} - \mathbf{k}$, $\mathbf{b} = 2\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$, $\mathbf{c} = 3\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ and $\mathbf{r} = \mathbf{i} + p\mathbf{j} - 3\mathbf{k}$ with respect to an origin O respectively. Find the value for p such that A, B, C and R points are coplanar. [20%]

   (b) If $\mathbf{a}$ and $\mathbf{b}$ are two vectors, define $\mathbf{a}.\mathbf{b}$ and $\mathbf{a} \times \mathbf{b}$. [15%]

   By using $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a}.\mathbf{c})\mathbf{b}$ - $(\mathbf{a}.\mathbf{b})\mathbf{c}$ [30%]

   deduce that

   i. $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) + \mathbf{b} \times (\mathbf{c} \times \mathbf{a}) + \mathbf{c} \times (\mathbf{a} \times \mathbf{b}) = 0$ [10%]

   ii. $(\mathbf{a} \times \mathbf{b}) \times (\mathbf{c} \times \mathbf{d}) = (\mathbf{a}, \mathbf{b}, \mathbf{d})\mathbf{c}$ - $(\mathbf{a}, \mathbf{b}, \mathbf{c})\mathbf{d}$, [05%]

   where $(\mathbf{a}.\mathbf{b}.\mathbf{c}) = \mathbf{a}(\mathbf{b} \times \mathbf{c})$

   (c) The position vectors of the points A, B and C are,

   $\mathbf{a} = \mathbf{i} + 2\mathbf{j} + 3\sqrt{2}\mathbf{k}$, $\mathbf{b} = 2\mathbf{i} + 2\mathbf{j} + 3\sqrt{2}\mathbf{k}$, $\mathbf{c} = 2\mathbf{i} + 3\mathbf{j} + 4\sqrt{2}\mathbf{k}$ respectively.

   i. Find an unit normal vector to the plane ABC. [15%]

   ii. Find an equation for plane ABC. [15%]

   iii. Find the angle between $\overrightarrow{AB}$ and $\overrightarrow{AC}$. [10%]

   iv. Find the area of the triangle ABC. [10%]

7. (a) A company makes a certain type of fan heater (called X heater) at each of its two factories F_1 and F_2. The factory F_1 produced one quarter and F_2 produce three quarter of the total output. X heaters are coloured either red or blue. One third of the X heaters produced at F_1 are red and seven-ninths of the X heaters produced at F_2 are red. A customer goes in to a shop and select a X heater at random. Show that the probability is $\frac{1}{3}$ that when he unpacks it, he will find that its blue. [20%]

   (b) Two shops A and B stocks heaters. Shop A has four and shop B has three. Find

   i. the probability that neither shop has a red X heaters. [20%]

   ii. the probability that there at least 3 red X heaters in shop A. [20%]

   (c) A discrete random variable X takes integer value from 1 to 7 inclusive with probabilities given by $p_{(X=x)} = \dfrac{1 + |4 - x|}{19}$.

   i. Find the distribution of the variable X. [20%]

   ii. Find $E(X)$ and $Var(X)$. [20%]

8. (a) Find a root of equation $x^3 - 5x + 3 = 0$ by using the Newton Raphson method.(for 3 decimal places) [40%]

(b) Tables give $\ln 2 = 0.693$. From the fact that $\ln 2 = \int_1^2 \frac{1}{x} dx$.
use the trapezoidal rule with four strips to get the approximation 0.697 and Simpson's rule with four strips to get the approximation 0.693 for $\ln 2$. [60%]

End