The Open University of Sri Lanka
B.Sc Degree Programme / Continuing Education Programme
Final Examination 2006 / 2007
AMU2185/AME4185-Numerical Methods I
Level 04-Applied Mathematics

Duration: Two and half ($2\frac{1}{2}$) hours

Date: 16/06/2007 Time: 10.00 a.m - 12.30 p.m.

Answer FOUR questions only

(1) (a) Briefly explain the following.
(i) Absolute error.
(ii) Relative error.
(iii)Truncation error.
(b) Complete the following computation.

$$\int_0^{\frac{1}{4}} e^{x^2}dx = \int_0^{\frac{1}{4}} (1+x^2+\frac{x^4}{2!}+\frac{x^6}{3!})dx$$

Compare your answer with the given value 0.2553074606 0f $\int_0^{\frac{1}{4}} e^{x^2}dx$.

What kind of error does arise in this case? Explain your answer.

(c) A can in the shape of a right circular cylinder has been constructed .The radius and the height of the can are $r = 2.000m$ and $h = 6.000m$ respectively. Find the volume v correct to the appropriate decimal places.

(2) (a) Let $f \in c[a,b]$ and suppose $f(a).f(b) < 0$. Show that the Method of Bisection generates a sequence $\{x^{(n)}\}$ approximating x^* with the property $\left|x^* - x^{(n)}\right| \le \frac{1}{2^n}(b-a)$, $n \ge 1$ where n is the number of iterations.

(b) Estimate the number of iterations that will be required to find the solution of $xe^x = 2$, using the interval [0, 1], correct to 4 decimal places by the Method of Bisection.

(c) Find the real root, correct to 4 decimal places of the equation $xe^x = 2$ in the interval [0, 1] by using the Method of Bisection .

(3) (a) (i) What is the geometric interpretation of the Newton's formula for solving $f(x)=0$.

(ii) With the usual notation, prove that the condition for convergence of the Newton's method is $|f(x^*)f''(x^*)| < [f'(x^*)]^2$; where x^* is the solution.

(b) Starting with $x_0 = 1.5$, use the Horner's scheme to find all approximate roots of $2x^4 - 10x^3 - 24x^2 + 152x - 158 = 0$ each to 4 decimal places.

(c) What are the advantages of using Newton's method than other methods?

(4) (a) Explain briefly how you would find the root of an equation by using simple iterative method .

(b) Discuss the convergence of the method.

(c) (i) Show that the equation $x^5 - 5x - 1 = 0$ has a real root in $[0.1, 0.2]$.

(ii) Derive a simple iterative scheme that can be expected to converge to the root.

(iii) Estimate the number of iterations that will be required to find an approximate root of $x^5 - 5x - 1 = 0$ to 4 decimal places .

(iv) Hence find an approximate root of the equation, up to 4 decimal places.

(d) Find the maximum error of this computation.

(5) (a) With usual notation obtain the followings.

(i) $\frac{1}{2}(E^{\frac{1}{2}} + E^{-\frac{1}{2}}) = \frac{2+\Delta}{2\sqrt{1+\Delta}} = \frac{2-\nabla}{2\sqrt{1-\nabla}}$

(ii) $\Delta = E\nabla$

(iii) $E^{\frac{1}{2}}\Delta = E^{-\frac{1}{2}}\nabla$

(b) Complete the following difference table

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
0	.....					
		.....				
5	.....		0.0118			
		0.1243		.....		
10	.....		.....		0.0012	
		.....		.....		-0.0001
15	.....		.....		.....	
		.....		0.0115		
20	.....		.....			
		.....				
25	1.7624					

(c) Using Newton's forward difference formula, find the interpolating polynomial which fits best for the given data.

(6) (a) Explain how the Lagrange interpolation polynomial $P(x)$ is found for the data set $(x_0, y_0), (x_1, y_1) \ldots\ldots\ldots, (x_n, y_n)$.

(b) With the usual notation, prove that the error of interpolation by Lagrange's method is $\frac{\pi(x)}{(n+1)!} f^{n+1}(c)$ where $c \in (x_0, x_n)$

(c) The following data gives the melting point of an alloy of Lead and Zinc, where t is the temperature in ${}^0C$ and P is the percentage of Lead in the alloy.

P	60	70	80	90
${}^0C$	231	255	281	309

Using Lagrange's interpolation formula, find the melting point of the alloy Containing 84 percentage of Lead.