The Open University of Sri Lanka
B.Sc. Degree Programme – Level 04
Final Examination 2006/2007
Applied Mathematics
AMU 2184/AME 4184 – Newtonian Mechanics

039

Duration :- Two and Half Hours.

Date :- 15-06-2007. **Time:- 10.00 a.m. – 12.30 p.m.**

Answer FOUR Questions only.

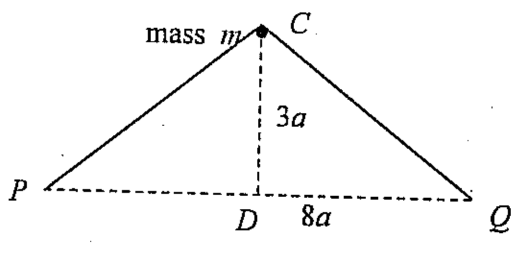
01. A particle moves under gravity in a medium which offers a resistance $kv^2/(a+y)$ per unit mass, where v is the speed of the particle, y is the height above a fixed point O and a and $k(\neq -1/2)$ are constants. If the particle is projected vertically upwards from O with speed u, show that it will come to rest instantaneously when $y = h$, where $(a+h)^{2k+1} = a^{2k+1}\{1 + u^2(2k+1)/2ag\}$.

Find also the speed with which the particle will return to O, and show that, if u^2 and ku^2 are both small, compared with ag, this speed will be approximately

$$u\left(1-\frac{ku^2}{2ag}\right)^{\frac{1}{2}}.$$

02. A tensile force T applied to a string of natural length l_0 and modulus of elasticity λ produces an extension ε. Show that the total energy stored in the string is $\frac{1}{2}\frac{\lambda\varepsilon^2}{l_0}$.

A catapult as shown projects a particle on a smooth horizontal surface. A light elastic string of natural length $8a$ and modulus of elasticity $2mg$ is attached to two points P and Q at a distance $8a$ apart on a horizontal table. A stone of mass m is attached to the mid-point D. The particle is drawn back a distance $3a$ to a point C from D and then released.



(i) What energy is stored in the elastic string when the mass is at point C?

(ii) Find the velocity of the stone when it reaches D.

(iii) The stone leaves the string when it reaches D, and is then acted upon by a resistance of magnitude mkv, where v is the speed of the particle and k is a constant. Show that it comes to rest at a distance $\frac{\sqrt{ga}}{k}$ from D.

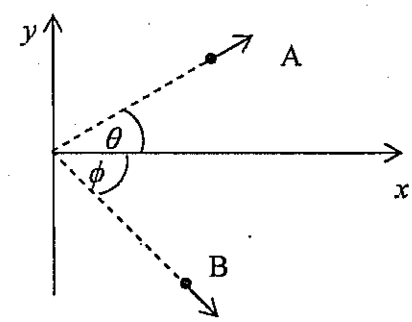
03. A system consists of n particles and if $\underline{F}_i^{ext}$ is the external force acting on the ith particle of mass m_i and $\underline{F}_{ij}$ is the internal force acting on the ith particle due to jth particle of mass mj.

(i) Show that the equation of motion for the system is given by $\underline{F}^{ext} = M\ddot{\underline{\bar{r}}}$ where $\underline{F}^{ext}$ is the resultant external force acting on the system, M is the total mass of the system and $\underline{\bar{r}}$ is position vector of the centre of mass.

(ii) If $\underline{F}^{ext} = \underline{0}$, show that the centre of mass of the system is moving with constant velocity.

(iii) Let $\underline{P}$ be the linear momentum of the system show that $\dot{\underline{P}} = M\ddot{\underline{\bar{r}}}$. Hence if $\underline{F}^{ext} = \underline{0}$, show that the linear momentum is conserved.

Two particles collide. Before the interaction particle A of mass 1kg has velocity $3\underline{i}\ \text{ms}^{-1}$ and particle B of mass 1kg is stationary. After the interaction both particles have speeds of 2ms^{-1} and move in the xy plane in the directions shown below in the diagram. Find the angles θ and ϕ.



04. Establish the formula $\underline{F}(t) = m(t)\frac{d\underline{v}}{dt} - \frac{dm}{dt}\underline{u}$ for the motion of a particle of varying mass $m(t)$ moving with velocity $\underline{v}$ under a force $\underline{F}(t)$, matter being added at a rate $\frac{dm}{dt}$ with velocity $\underline{u}$ relative to the particle.

A raindrop is observed at time $t = 0$ when it has mass m and downward velocity u. As it falls under gravity its mass increases by condensation at a constant rate λ and a resisting force acts on it, proportional to its speed and equal to λv when the speed is v. Show that

$\frac{d}{dt}(M^2 v) = M^2 g$ where $M = m + \lambda t$, and find the speed of the raindrop at time t.

05. A particle P, of mass m, is moving under the action of a force F in a plane that contains the point O. The force acts along PO and has magnitude $m\dfrac{V^2a^3}{r^4}$, where V and a are constants and $r = OP$.

When $t = 0$, P is projected from a point at a distance a from O with speed $\dfrac{V}{\sqrt{6}}$ in a direction perpendicular to OP. Given θ is the angle between the line OP at time t and its position when $t = 0$ show that:

(a) $r^2\dot{\theta} = a\dfrac{V}{\sqrt{6}}$

(b) $\ddot{r} = \dfrac{V^2a^2}{6r^4}(r - 6a)$

(c) $\dot{r}^2 = \dfrac{V^2(4a^3 - a^2r - 3r^3)}{6r^3}$.

Show further that, in the subsequent motion, r never exceeds a.

Find the distance of the particle from O when the speed is V.

06. With usual notation derive the equation of the central orbit $\dfrac{d^2u}{d\theta^2} + u = \dfrac{P}{h^2u^2}$ when $u = \dfrac{1}{r}$ and P is the central force per unit mass. A minor body of mass m is moving with respect to a major body of mass M. The force between two bodies satisfies Newton's law of Gravitation. Show that the equation of the central orbit of m relative to M is given by $u = \dfrac{1}{r} = C\cos\theta + \dfrac{GM}{h^2}$, where C is an arbitrary constant.

Show further that the path is an ellipse, parabola or hyperbola according as $C \lesseqqgtr \dfrac{GM}{h^2}$.

If V_e the escape velocity at apogee ($r = r_0$) for a parabolic path then show that $V_e = \sqrt{\dfrac{2GM}{r_0}}$.

If V_0 is the escape velocity needed for hyperbolic path then show that $V_0 > V_e$ and

If V_0 is the escape velocity needed for elliptic path then show that $V_e > V_0 > \dfrac{V_e}{\sqrt{2}}$.