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THE OPEN UNIVERSITY OF SRI LANKA
Faculty of Engineering Technology
Department of Mathematics & Philosophy of Engineering

Bachelor of Technology Honors in Engineering

Final Examination (2016/2017)
MPZ3132: Engineering Mathematics IB

Date: 13th November 2017 (Monday) **Time: 9:30 am – 12:30 pm**

Instruction:

- **Answer only five questions.**
- **Number of pages in the paper -06.**
- **All symbols are in standard notation and state any assumption that you made.**

Important integrals

- $\int e^{ax} sinbx dx = \frac{e^{ax}}{a^2+b^2}(asinbx - bcosbx)$
- $\int e^{ax} cosbx dx = \frac{e^{ax}}{a^2+b^2}(acosbx + bsinbx)$
- $\int f(x)e^{ax}\, dx = \frac{1}{a}f(x)e^{ax} - \frac{1}{a^2}f^1(x)e^{ax} + \frac{1}{a^3}f^2(x)e^{ax} \ldots\ldots$ [Sign alternate $(+ - + - + \cdots)$]
- $\int f(x)cosax\, dx = \frac{1}{a}f(x)sinax + \frac{1}{a^2}f^1(x)cosax - \frac{1}{a^3}f^2(x)sinax - \cdots$ [Sign alternate in pairs $- - + + - - + + - - \cdots$)]
- $\int f(x)sinax\, dx = -\frac{1}{a}f(x)cosax + \frac{1}{a^2}f^1(x)sinax + \frac{1}{a^3}f^2(x)cosax - \cdots$ [Sign alternate in pairs after the first term $(+ + - - + + - - + + - - \cdots)$]
- $\int secax\, dx = \frac{1}{a}ln|secax + tanax|$

Q1.

I. Write down the Fourier series expansion for the function $f(x)$ which satisfies the Dirichlet's conditions. [20%]

II. Expand $f(x) = x$, in a half range $0 < x < 2$ as a: [50%]

a) Sin series,
b) $Cosine$ series.
c) Applying the Parseval's identity formula to the Fourier series of part (II) (b), deduce that $\sum_{i=0}^{\infty} \frac{1}{n^4} = \frac{\pi^4}{90}$.

III. Find the Fourier series representation of the function with period 4, defined by

$$f(x) = \begin{cases} -1, & -2 \leq x \leq 0, \\ 2, & 0 < x \leq 2. \end{cases}$$ [30%]

Q2.

I.

a) Define the Taylor series of $f(x)$ about $x = a$. [05%]
b) Find the Taylor series of the following functions at the each of the following given value of a:

$f(x) = 2x - 3x^2$ at $a = -2$, and

$g(x) = e^{-2x}$ at $a = \frac{1}{2}$. [25%]

II.

a) Find the Taylor series of $h(x) = \frac{1}{1+x}$ about $x = 0$.

Hence, find the Taylor series of $(1-x)^{-1}$ and $(1+x^2)^{-1}$ about $x = 0$. [25%]

b) Using the above part (a), find the Taylor series for $tan^{-1}x$ about $x = 0$. [10%]
c) Find the 4th order Taylor polynomial for the following function about $x = 0$.

$$p(x) = \frac{x+7}{x^2-x-6}$$ [35%]

(Hint: First find the Partial fraction decomposition of the given function, and use the Tylor series expansion)

Q3.

I. Solve the following differential equation:
$(D^3 + 3D^2 + 3D + 1)y = e^{-x}$ [25%]

II. Using a suitable trial function:

a) Find a particular integral for the following differential equation:

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = \sin 2x.$$

b) Find the general solution of the above differential equation [35%]

III. Prove that

$$\frac{1}{D+\alpha} f(x) = e^{-\alpha x} \frac{1}{D} e^{\alpha x} f(x).$$

Find a particular integral of the following differential equation:

$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 5\cos 3x$. Hence, find the general solution. [40%]

Q4.

I. Define the "Laplace transformation" of $F(t)$. [05%]

II. Find the "Laplace transformation" of each of the following function: [45%]

a) $3t^2 - 4e^t + 5\sin 2t - 6\cos 3t$,

b) te^t,

c) $(t^2 + 4)e^{2t} - e^{-t}\cos t$.

III. Find the inverse "Laplace transformation" of each of the following: [20%]

a) $\frac{s+1}{s^2-16}$

b) $\frac{1}{(s+1)(s^2+1)}$.

IV. Using the "Laplace transformation", solve the following differential equation:
$y' + y = 5\sin x, \quad y(0) = 3$ [30%]

Q5.

I. Define the Poisson distribution, and obtain its expectation and variance. [25%]

II. The number of calls coming per minute into a hotel reservation center is Poisson random variable with mean 3.

a) Find the probability that no calls come in a given 1 minute period.

b) Find the probability that at least 3 calls will arrive in a given 1 minute period. [25%]

III. Suppose that we are told that the heights of adult males in a particular region of the world are normally distributed with mean of 70 inches and standard deviation of 2 inches. [50%]

a) Approximately what proportion of adult males are taller than 73 inches?

b) What proportion of adult males are between 72 and 73 inches?

c) What height corresponds to the point where 20% of all adult males are greater than this height?

d) What height corresponds to the point where 20% of all males are less than this height?

Q6.

I.

a) Shade the region corresponding to $|Z + i| \geq |Z - 2|$ in the Argand plane. **[20%]**

b) Sketch the region in the Argand diagram define by

$$6 \leq Re[(2 - 3i)Z] < 12 \quad \text{and} \quad Re\, Z . Im\, Z \geq 0 \ .$$ **[20%]**

c) Express $\log(1 - i)$ in the form of $x + iy$, where x and y are real numbers. **[15%]**

II.

a) Find the unit tangent vector and unit normal vector at $t = 2$ on the curve $x = t^2 - 4t,\ y = 4t - 3,\ Z = 2t^2 - 6t,$ where t is a variable. [30%]

b) A particle M moves with velocity $2\underset{\sim}{i} - 3\underset{\sim}{j}$ from point (4, 5). At the same instant a particle N, moving in the same plane with velocity $4\underset{\sim}{i} + \underset{\sim}{j}$, passes through a point (2, -1). Determine whether the particle M and N are collide or not. [15%]

Q7.

I. Find the moment of inertia of the following bodies about the given axis.

a) A uniform rod AB of mass M_1 and length $2a$, the axis perpendicular to the rod passing through the midpoint of the rod. [15%]

b) Three uniform rod each of length $\boldsymbol{a}$ and mass $\boldsymbol{m}$ are rigidly jointed at their ends to form a triangular framework. Find the moment of inertia of the framework about an axis passing through the midpoints of two of its sides. [30%]

II.

a) A uniform circular disc of mass $\boldsymbol{m}$ and radius $\boldsymbol{a}$. Find the moments of inertia each of the following case: [25%]

(α). about the axis perpendicular to the plane of the disc passing through the center,

(β). about the axis passing through the diameter,

(γ). about the axis passing through the point in circumference of the disc parallel to the diameter.

b) Find the moment of inertia of a solid circular cylinder of radius $\boldsymbol{a}$, length $\boldsymbol{h}$ and mass $\boldsymbol{M}$ about a diameter of an end face. [30%]

Q8.

I. A uniform rectangular lamina $PQRS$ is immersed vertically in a homogeneous liquid such that PQ and RS are horizontal. The distance to PQ and RS from the surface of the liquid are a and b respectively. Prove that the center of pressure of the lamina is at the distance

$\frac{2}{3}\left(\frac{a^2+ab+b^2}{a+b}\right)$ from the free liquid surface. **[40%]**

II. A semi circular lamina of radius a and center O is immersed in a homogenous liquid with its plane vertical and the straight edge on the surface. Show that the center of the lamina is at depth $\frac{3\pi a}{16}$ from O. **[60%]**